On the Use of Teletraffic Theory in Power Distribution Systems

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Teletraffic theory – a simple example

- Suppose *n* independent ON-OFF traffic sources share a link
- The probability that a source is in state ON is *p*
- The traffic bit rate in state ON is *r*

What is the probability that the bit rate of the aggregate traffic is greater than *kr*?

$$\sum_{i=k+1}^{n} \binom{n}{i} p^i (1-p)^{n-i}$$

Teletraffic theory – a simple example

- Suppose n independent ON-OFF traffic sources share a link
- The probability that a source is in state ON is p
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What is the probability that the bit rate of the aggregate traffic is greater than kr?

What is the minimum value of k for which this probability is less than ε?

How can we approximate this probability for large values of n and k?

Teletraffic theory – another example

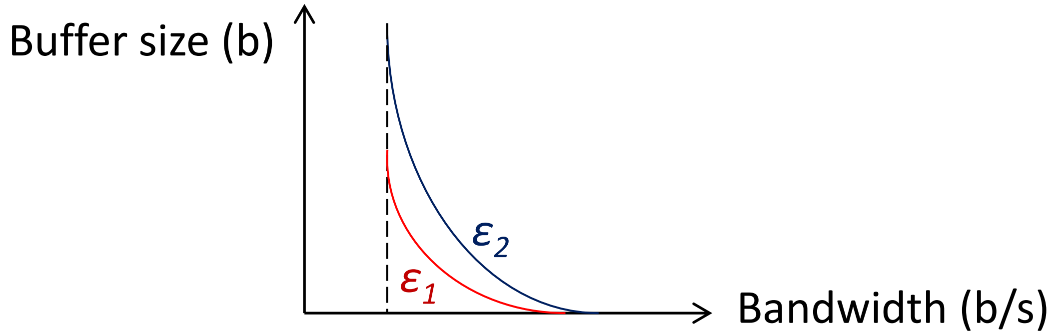
- Suppose we have n independent ON-OFF traffic sources
- The probability of being in state ON is p
- The traffic bit rate in state ON is r
- A router serves the aggregate traffic at rate C
- This router has a buffer with capacity B

What is the probability that an arriving bit/packet finds the buffer full?

Teletraffic theory – another example

- Suppose we have *n* independent ON-OFF traffic sources
- The probability of being in state ON is *p*
- The traffic bit rate in state ON is *r*
- A router serves the aggregate traffic at rate *C*
- This router has a buffer with capacity *B*

What (*C*,*B*) pairs can be chosen such that this probability is less than *ε*?

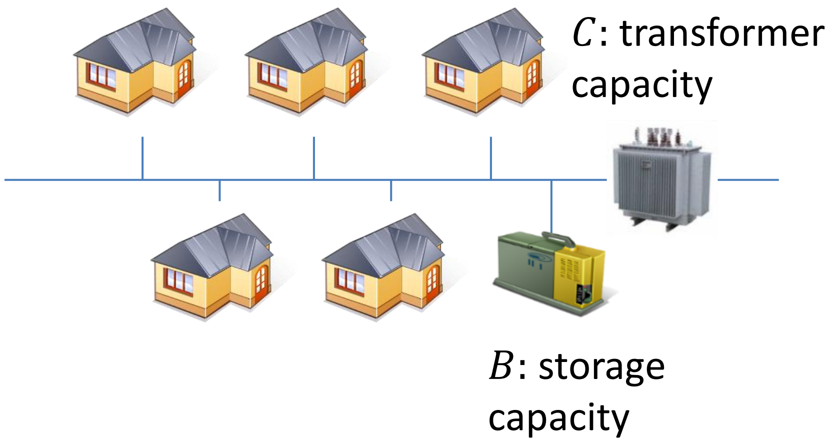


Teletraffic theory allows us to dimension a telecommunication network with:

- Heterogeneous traffic sources
- Shared transmission facilities
- Specific quality of service requirements

A power distribution network consists of:

- Stochastic electricity demand
- Shared lines, transformers, and storage



A certain level of “reliability” is guaranteed

Reliability of the grid

- Loss of load probability is one measure of reliability
- Loss of load *may* happen when a transformer is *overloaded*

Goal:

To size transformers, storage, and renewable energy generators in power distribution networks using teletraffic theory originally developed to size links, routers, and buffers in telecommunication networks

Sizing for the peak – the current practice

- Demand uncertainty is low
- The peak demand can be forecasted with high accuracy
- Optimal transformer sizing can be found using the load profile of the peak days of year

A new sizing approach is needed

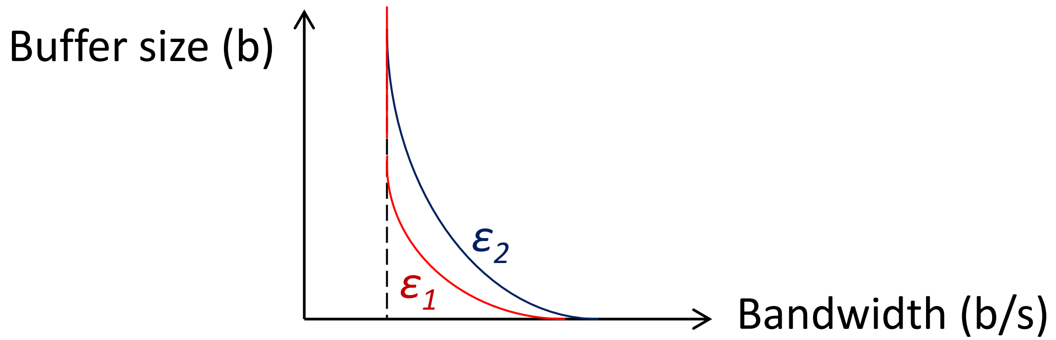
- The demand uncertainty will increase
- Storage will be installed in the distribution network to smooth out variations in demand
- Transformers can be sized closer to the average demand

Contributions

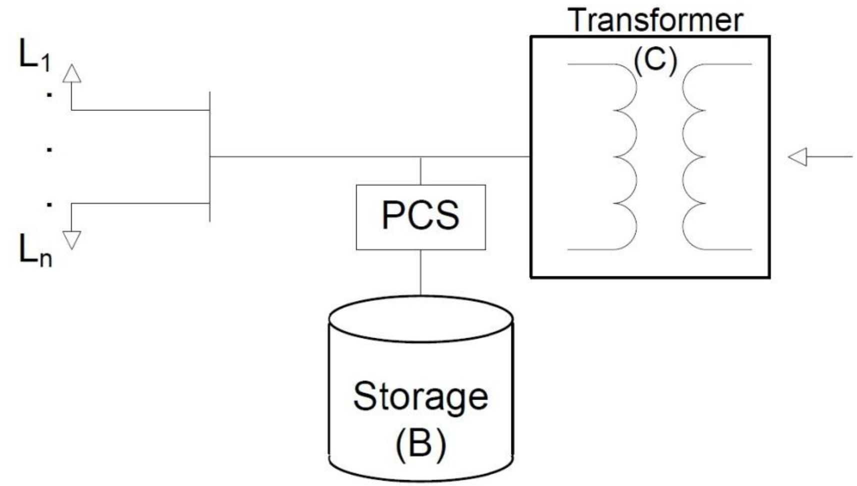
- Modelling the distribution network as a fluid queueing system
- Applying teletraffic theory to size distribution transformers
- Validation of the proposed sizing approach using actual and synthetic load traces

Queueing Models & teletraffic-based sizing

The buffer/bandwidth trade-off



Observations:



- Storage is a buffer
- A transformer charges storage as traffic sources fill the buffer
- Loads discharge storage as a router empties the buffer
- The loss of load probability is similar to the packet loss probability

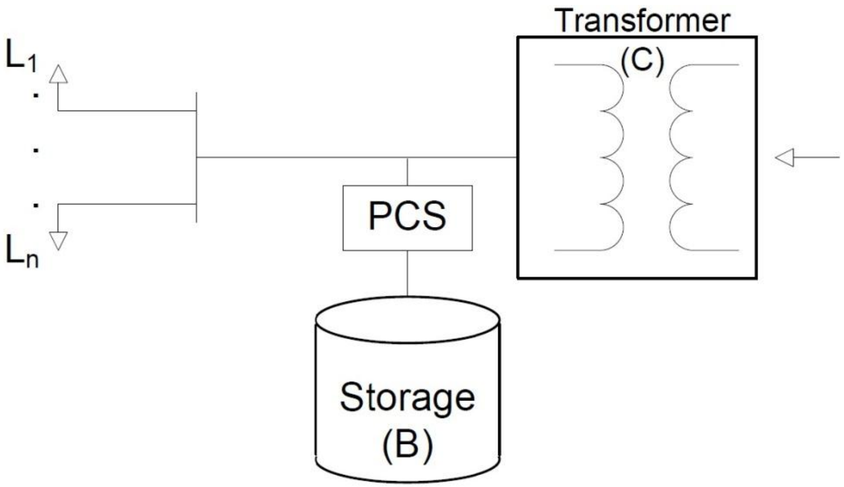
A fluid queueing model can be associated to a radial power distribution network

- A fluid queue with constant arrival rate and arbitrary service rate

- We want to quantify the buffer (storage) underflow probability in this model

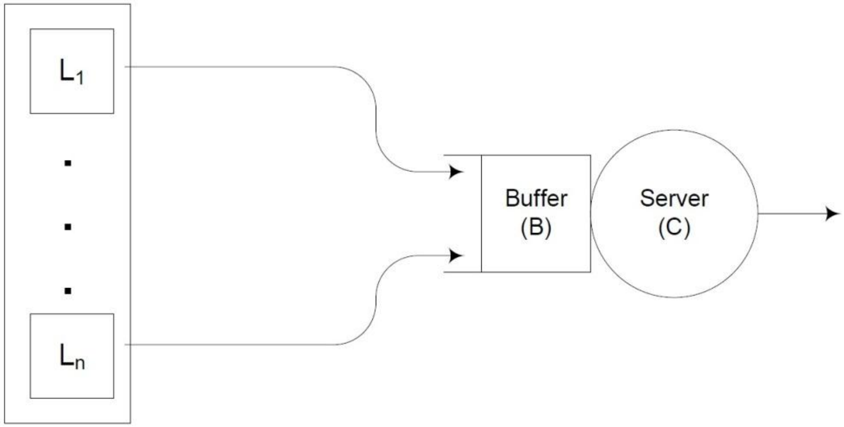
- Unfortunately teletraffic analysis does not deal with this question

A dual queueing model



ε: storage underflow probability

The Equivalence Theorem



buffer overflow probability ≅ *ε*

Transformer Sizing

A Case Study

Resource allocation and effective bandwidth

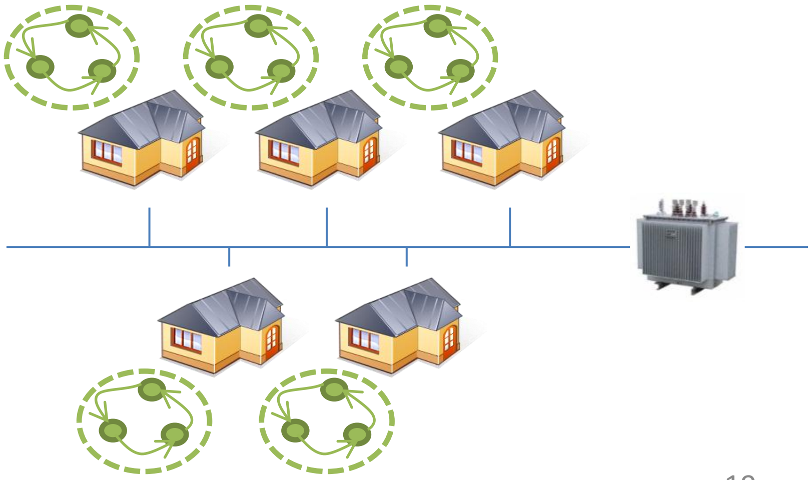
- It is shown that the overflow probability is defined in terms of the aggregate *effective bandwidth* of homes supplied by a transformer

- Effective bandwidth of a stochastic source represents the amount of resource that should be reserved for it

- Computing effective bandwidth requires modelling the electricity demand of each home

Load Modelling

- A neighbourhood of 20 homes
- Classified into 4 classes
- Busy hour electricity demands of homes in each class are used to construct the Markov model of this class
- The aggregate effective bandwidth of the neighbourhood is the sum of effective bandwidths of all homes

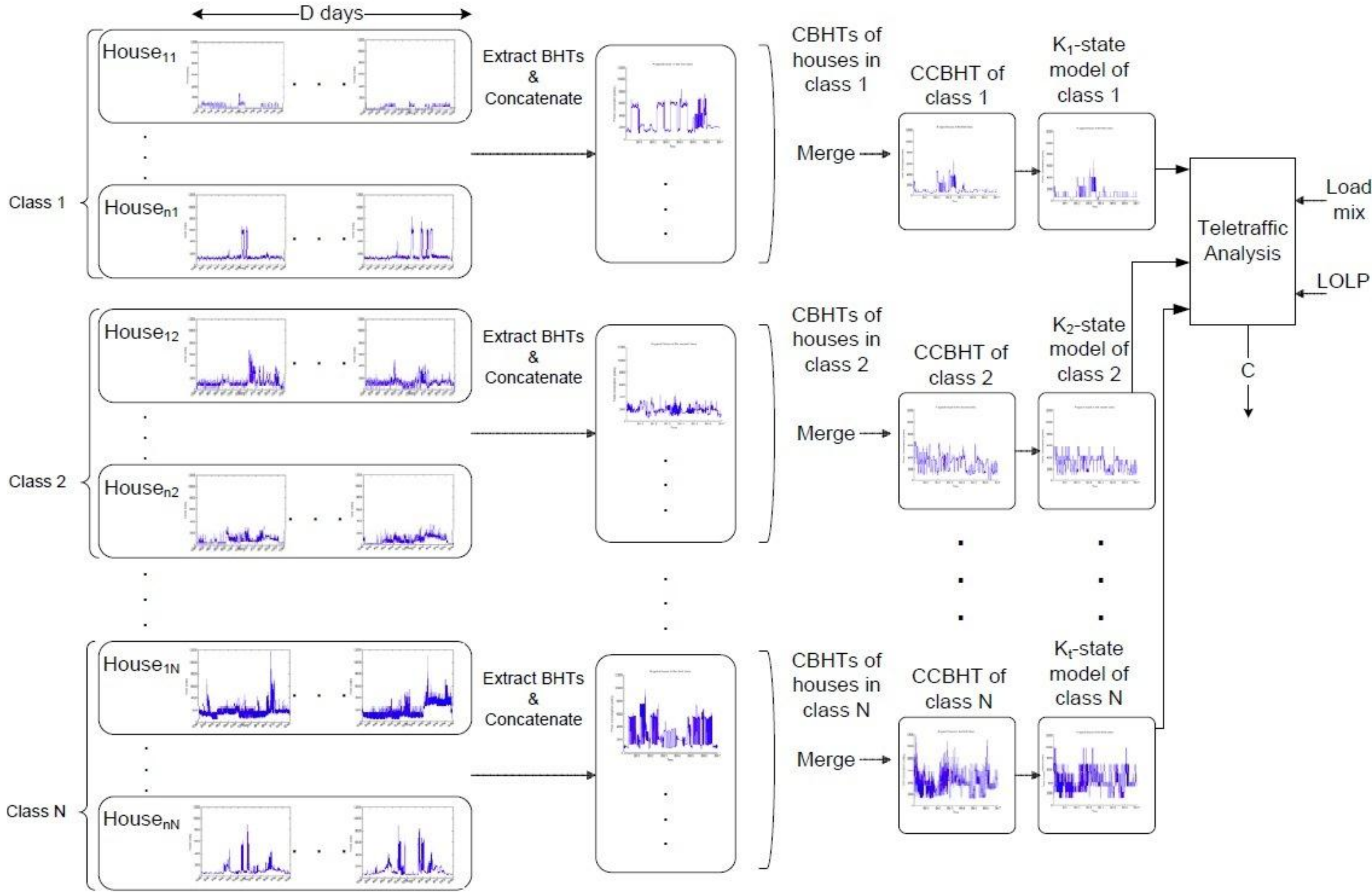


Teletraffic-based Sizing of Power Distribution Networks

Measurement

Modelling

Analysis



Results of the Teletraffic-based Sizing Approach

- The transformer capacity computed for a neighbourhood given the industry standard loss of load probability is 107 kVA

- The utility guideline based on decades of field experience recommends a 100 kVA transformer for the same neighbourhood

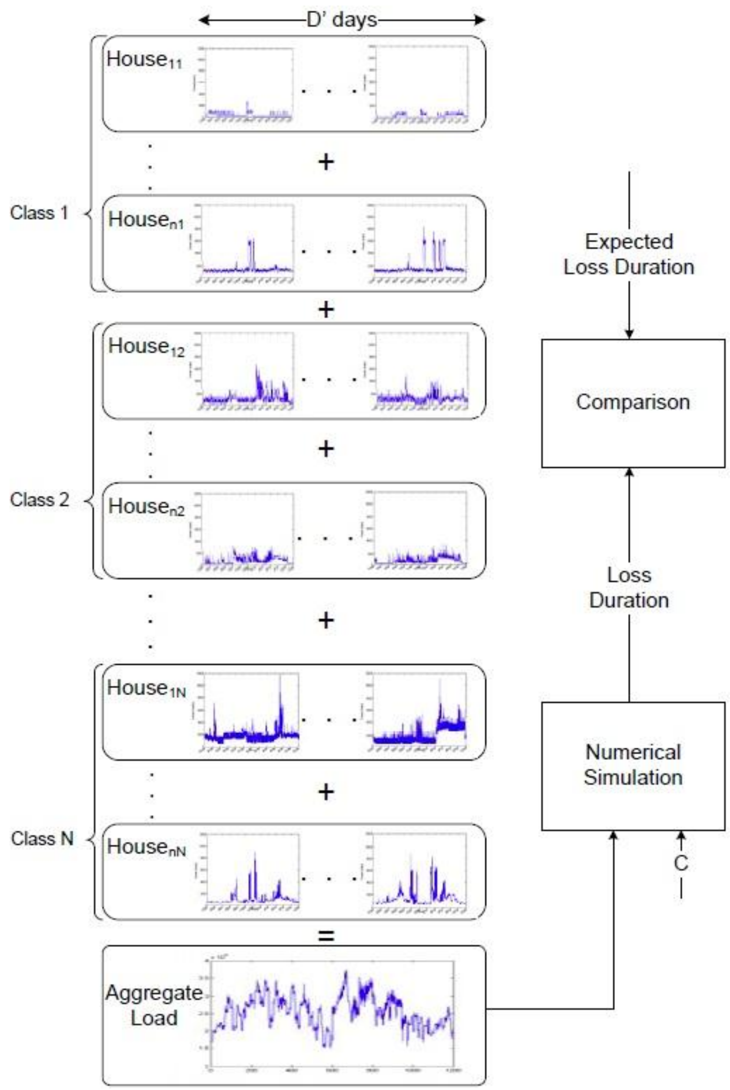
Validation

Measurement

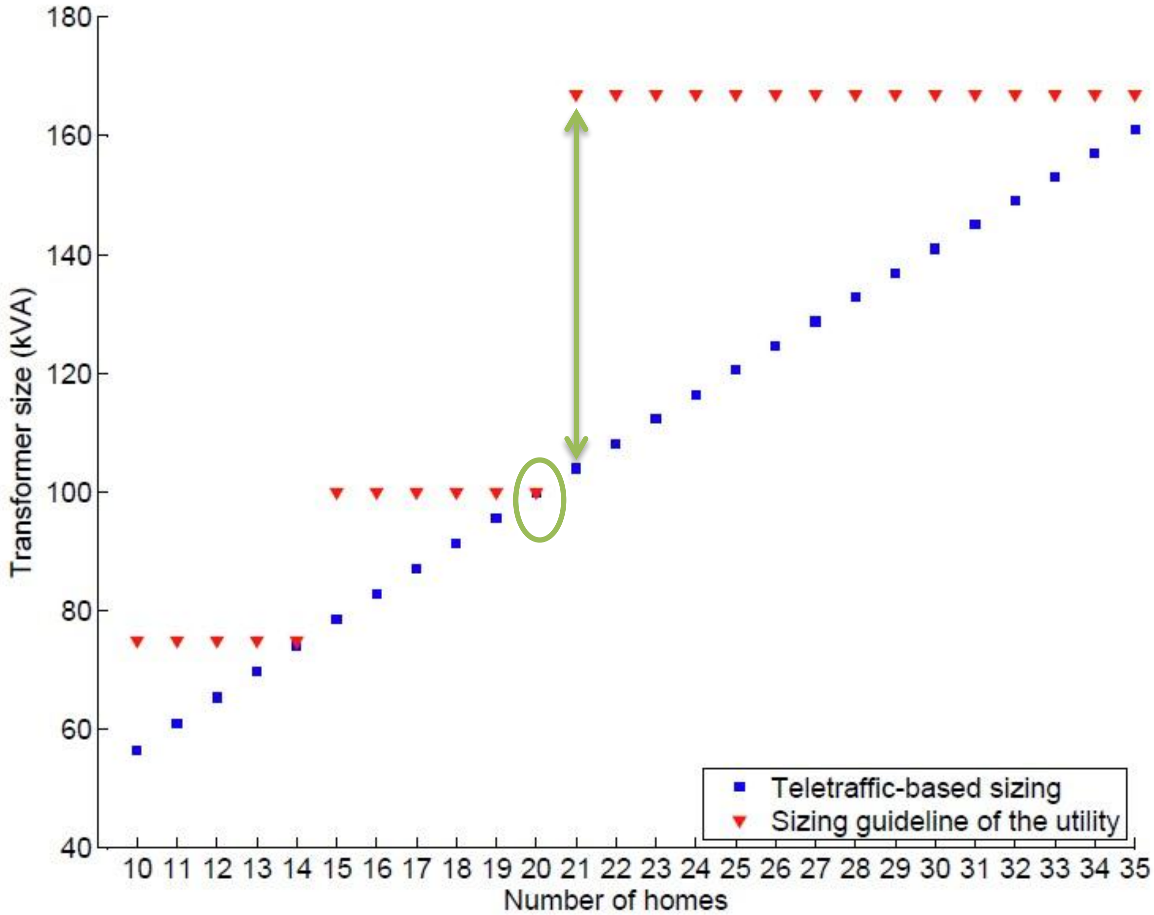
Modelling

Analysis

Validation



Comparison of the teletraffic-based sizing with the sizing guideline of a utility



a homogeneous load mix

Conclusions

- A distribution network can be modelled as a fluid queueing system
- Teletraffic theory can be applied to size
  - Transformers
  - Transformers and storage jointly