Resource Provisioning in the Electrical Grid

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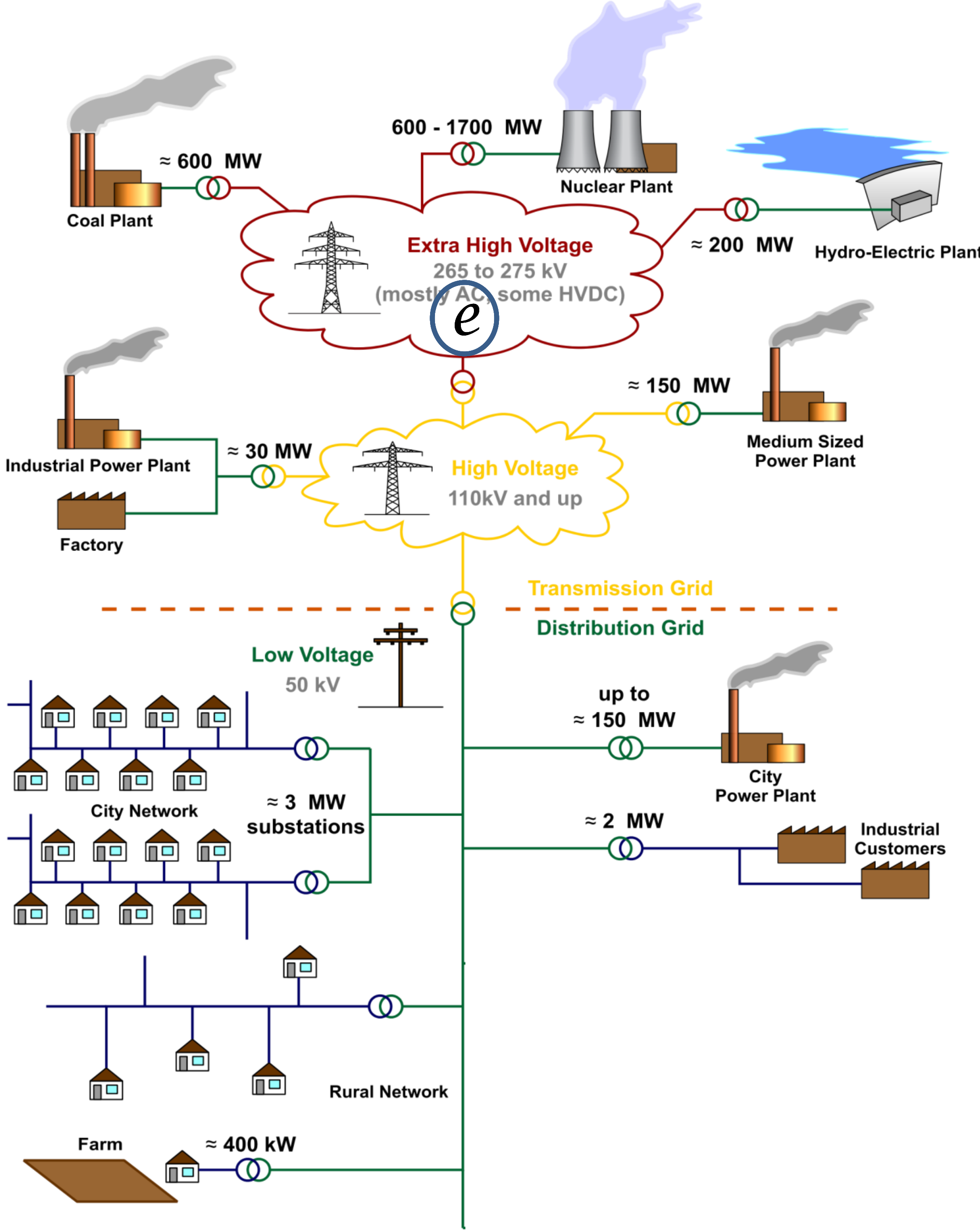
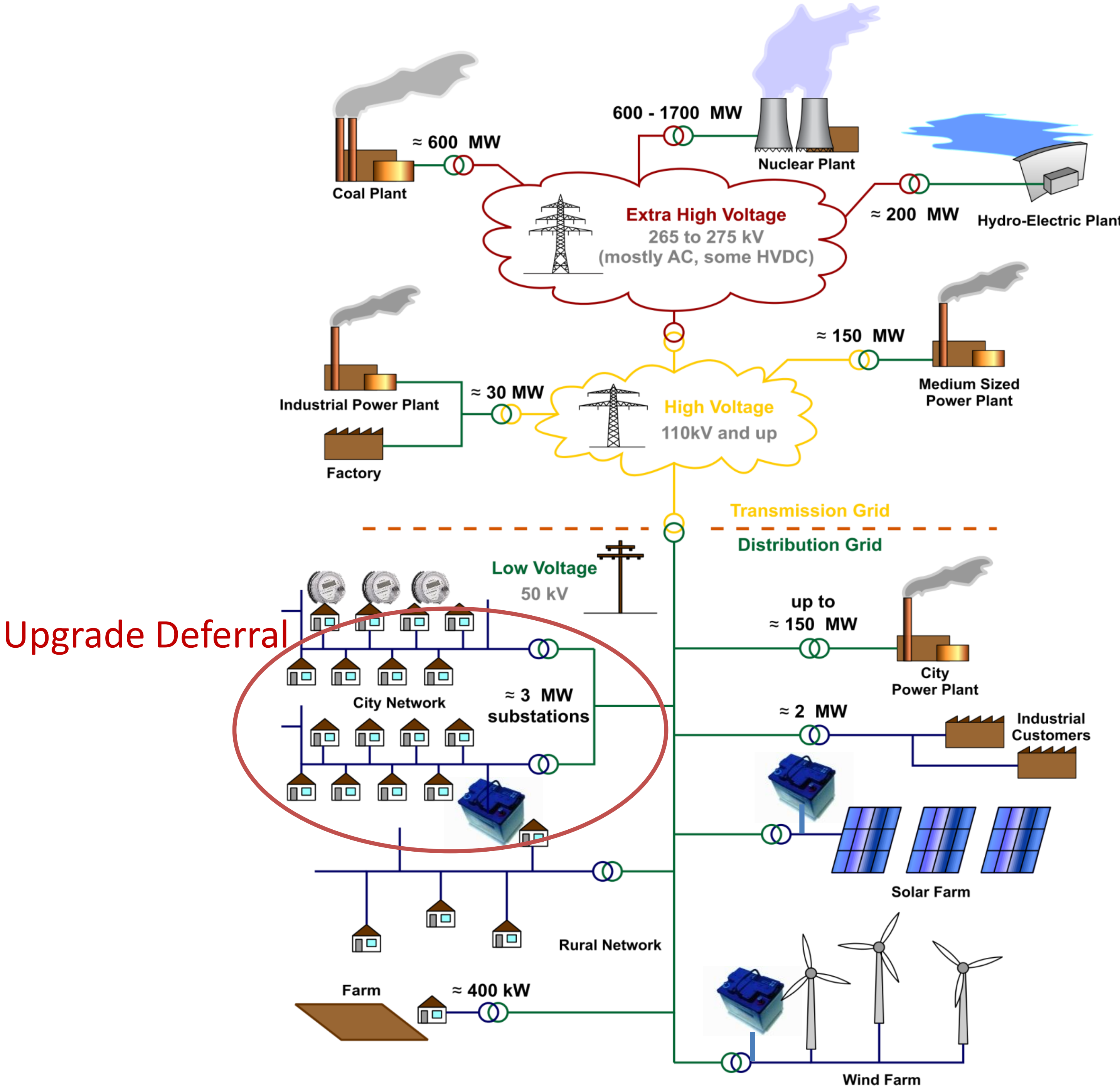
University of Waterloo

Master's Thesis Presentation
April 2010


≈ 600 MW
Coal Plant
600 - 1700 MW
Nuclear Plant
≈ 200 MW
Hydro-Electric Plant
Extra High Voltage
265 to 275 kV
(mostly AC, some HVDC)
e
≈ 150 MW
Medium Sized
Power Plant
Industrial Power Plant
≈ 30 MW
Factory
High Voltage
110kV and up
Transmission Grid
Distribution Grid
Low Voltage
50 kV
up to
≈ 150 MW
City
Power Plant
City Network
≈ 3 MW
substations
≈ 2 MW
Industrial
Customers
Rural Network
Farm
≈ 400 kW



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≈ 2 MW
Industrial
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Solar Farm
Rural Network
Farm
≈ 400 kW
Wind Farm

Key Contributions

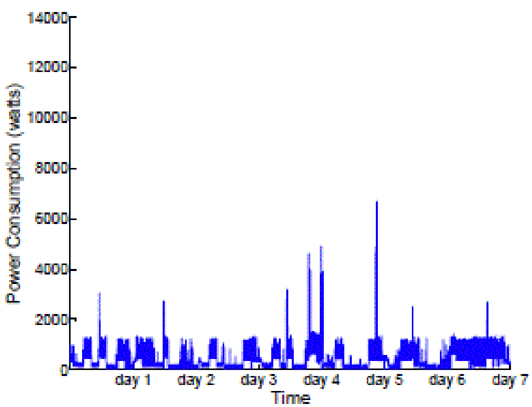
- Modeled the home load
- Showed the electricity grid is equivalent to a queuing network
- Developed a guideline for sizing the electrical grid

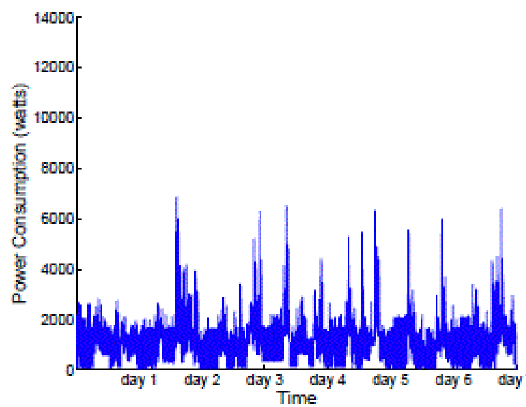
Outline

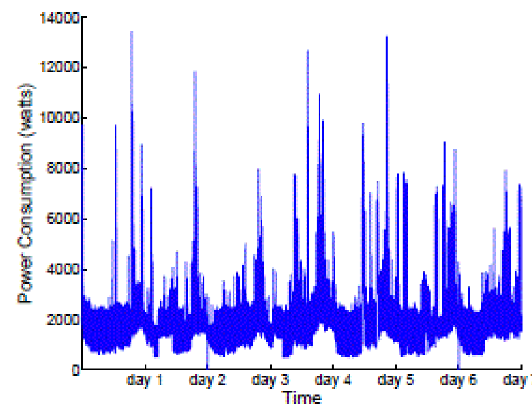
- Load Measurement
- Load Modeling
- Sizing the Grid
  - System Model
  - Equivalence
  - Effective Bandwidth
  - Teletraffic-based Sizing
  - Validation
- Conclusion

Load Measurement: Testbed and Classification

9 Houses
3 Classes







Load Modeling

- Why should we model a house as a Markovian source?
  - Aggregate load is the superposition of a finite set of on-off loads from individual appliances
  - Enables us to use queuing theory and teletraffic analysis

Markov Modeling

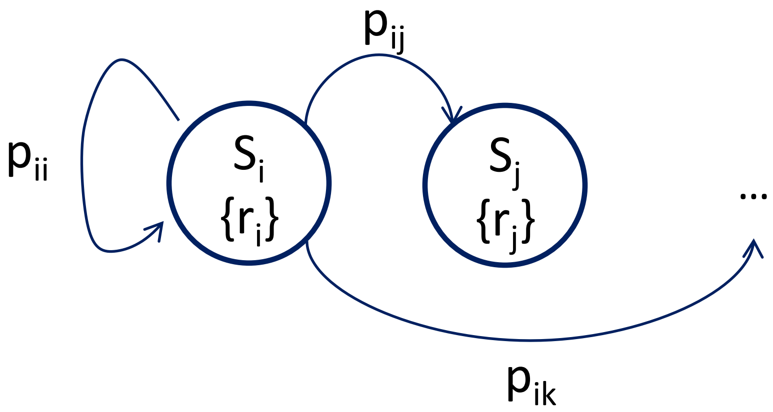
- Definition of a Markovian source
- How many models are needed?
- Choosing Markov states
- How many Markov states should a model have?

Definition of a Markovian Source

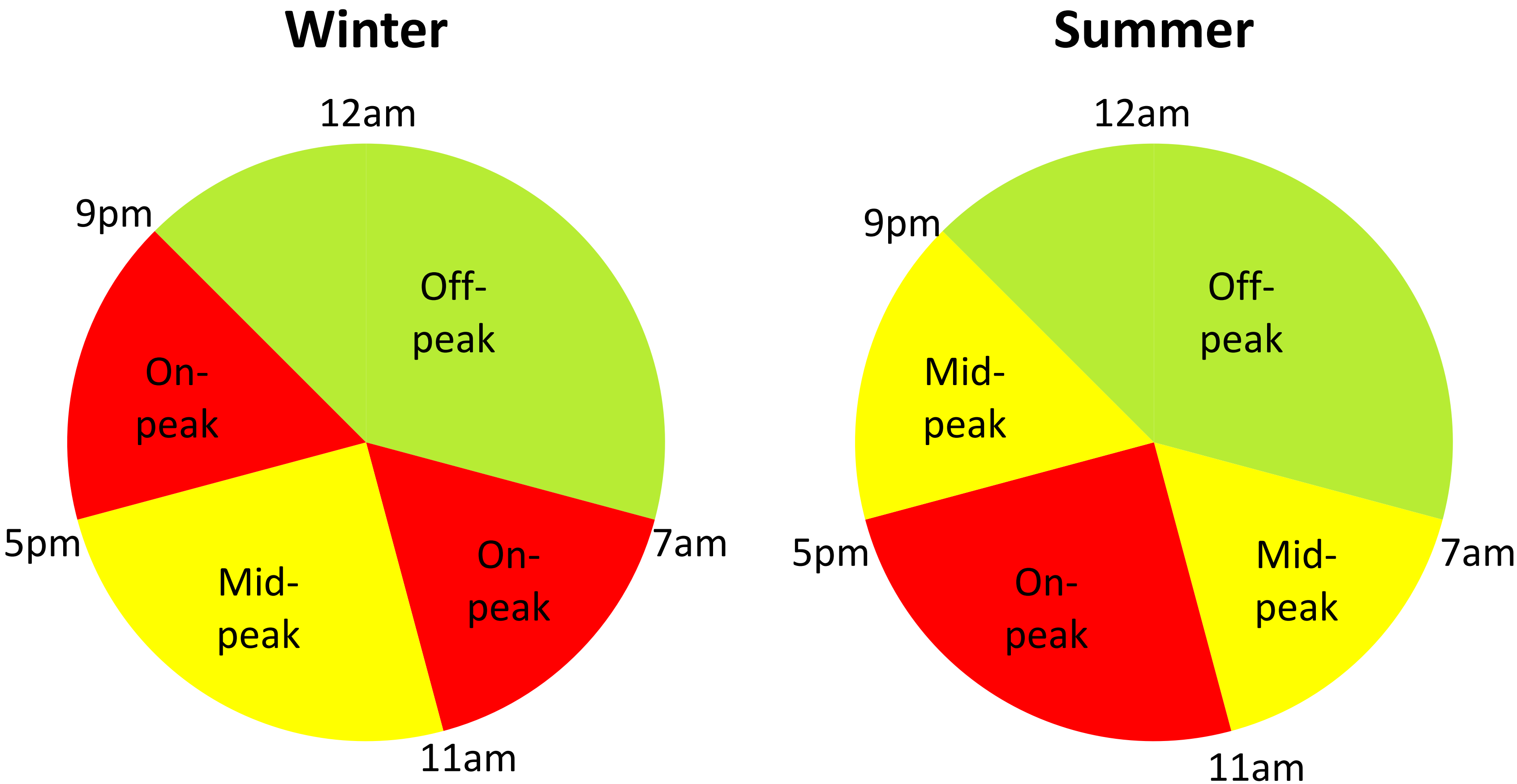
- Governed by a discrete-time Markov chain

$$P = \begin{bmatrix} p_{11} & \cdots & p_{1k} \\ \vdots & \ddots & \vdots \\ p_{k1} & \cdots & p_{kk} \end{bmatrix}$$

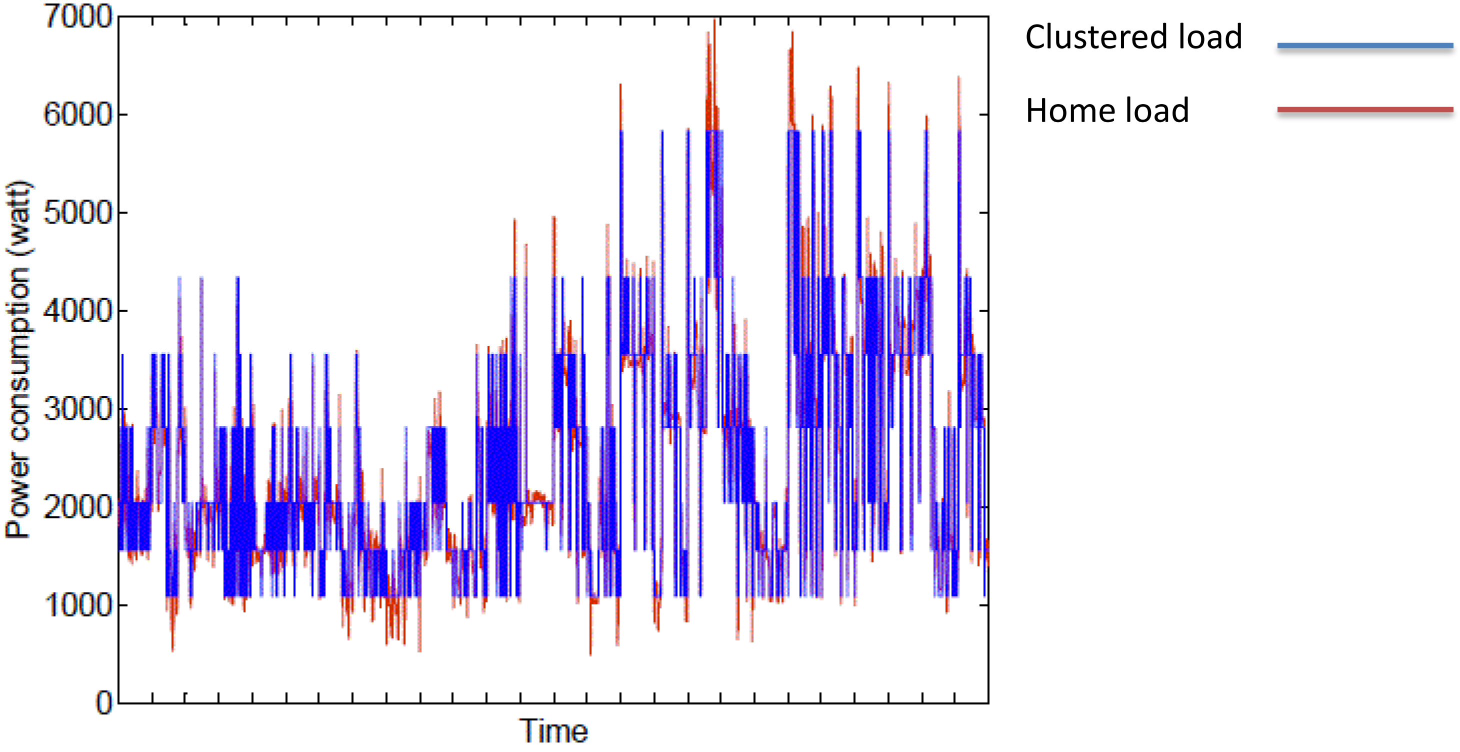
$$R = \begin{bmatrix} r_1 & \cdots & r_k \end{bmatrix}$$



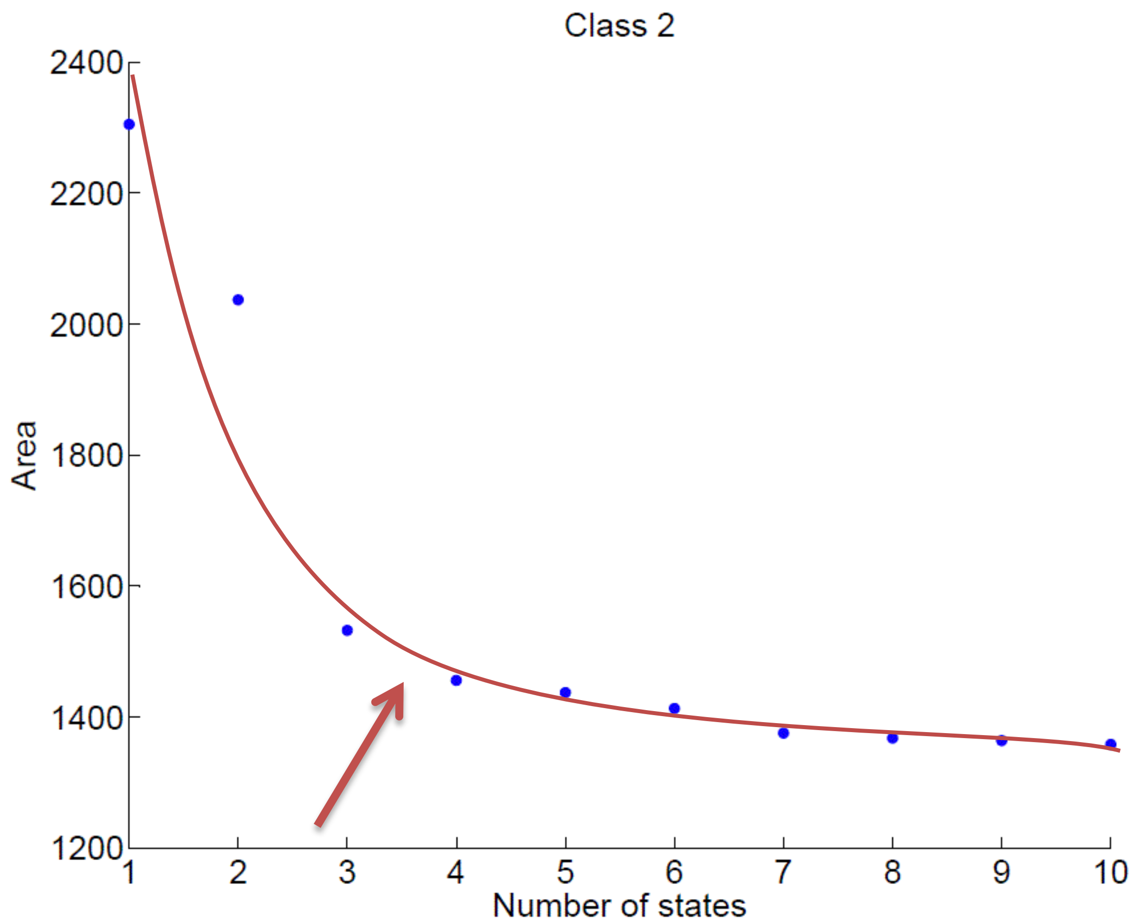
How Many Models are Needed?



Choosing Markov States

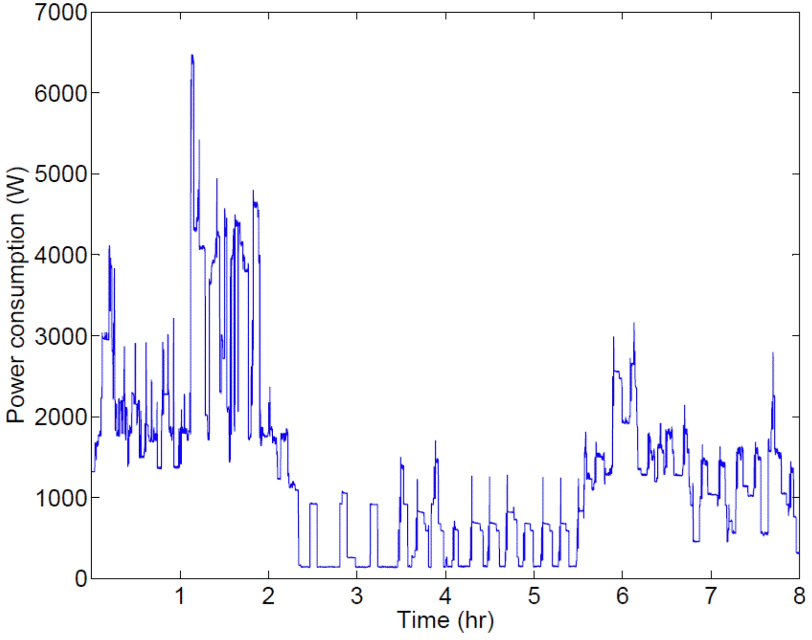


How Many Markov States Should a Model Have?

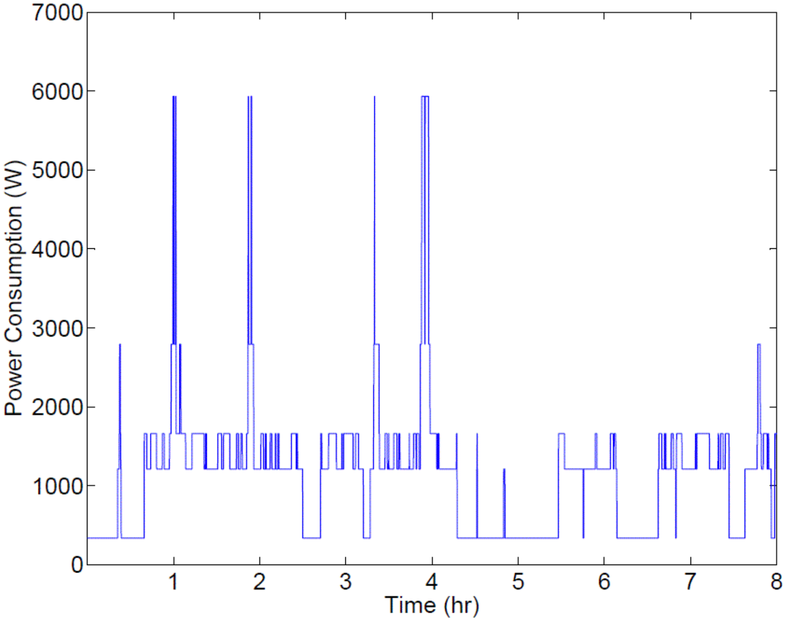


Load Modeling - Validation

Ground truth

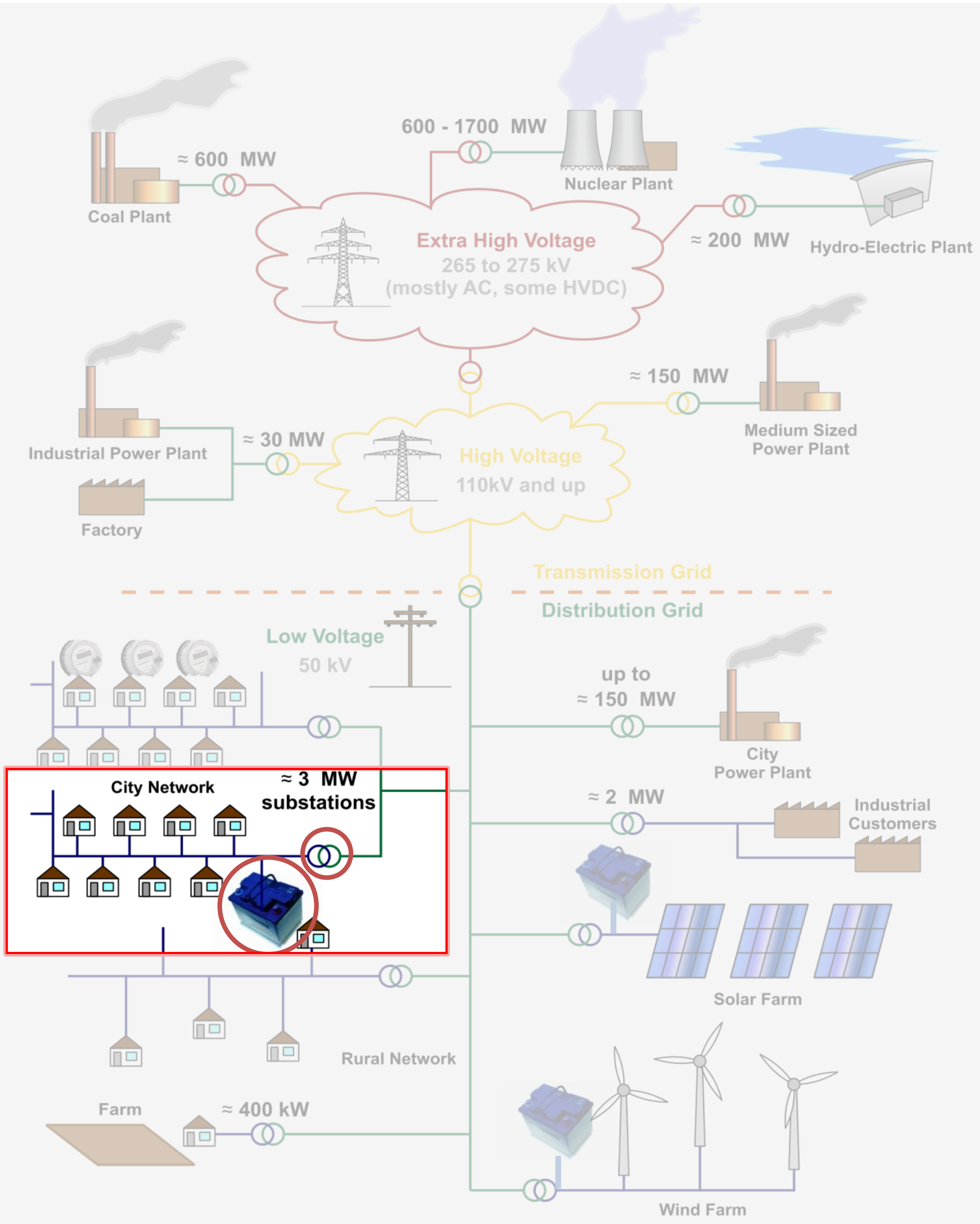


Synthesized trace



$$P = \begin{bmatrix} 0.90264 & 0.00192 & 0.01080 & 0.00834 & 0.07630 \\ 0.00103 & 0.97198 & 0.00006 & 0.02210 & 0.00481 \\ 0.06325 & 0.00110 & 0.91737 & 0.00183 & 0.01645 \\ 0.00336 & 0.01929 & 0.00028 & 0.94352 & 0.03355 \\ 0.02448 & 0.00166 & 0.00108 & 0.03038 & 0.94239 \end{bmatrix}$$

$$R = \begin{bmatrix} 2252 & 500 & 4355 & 1077 & 1614 \end{bmatrix}$$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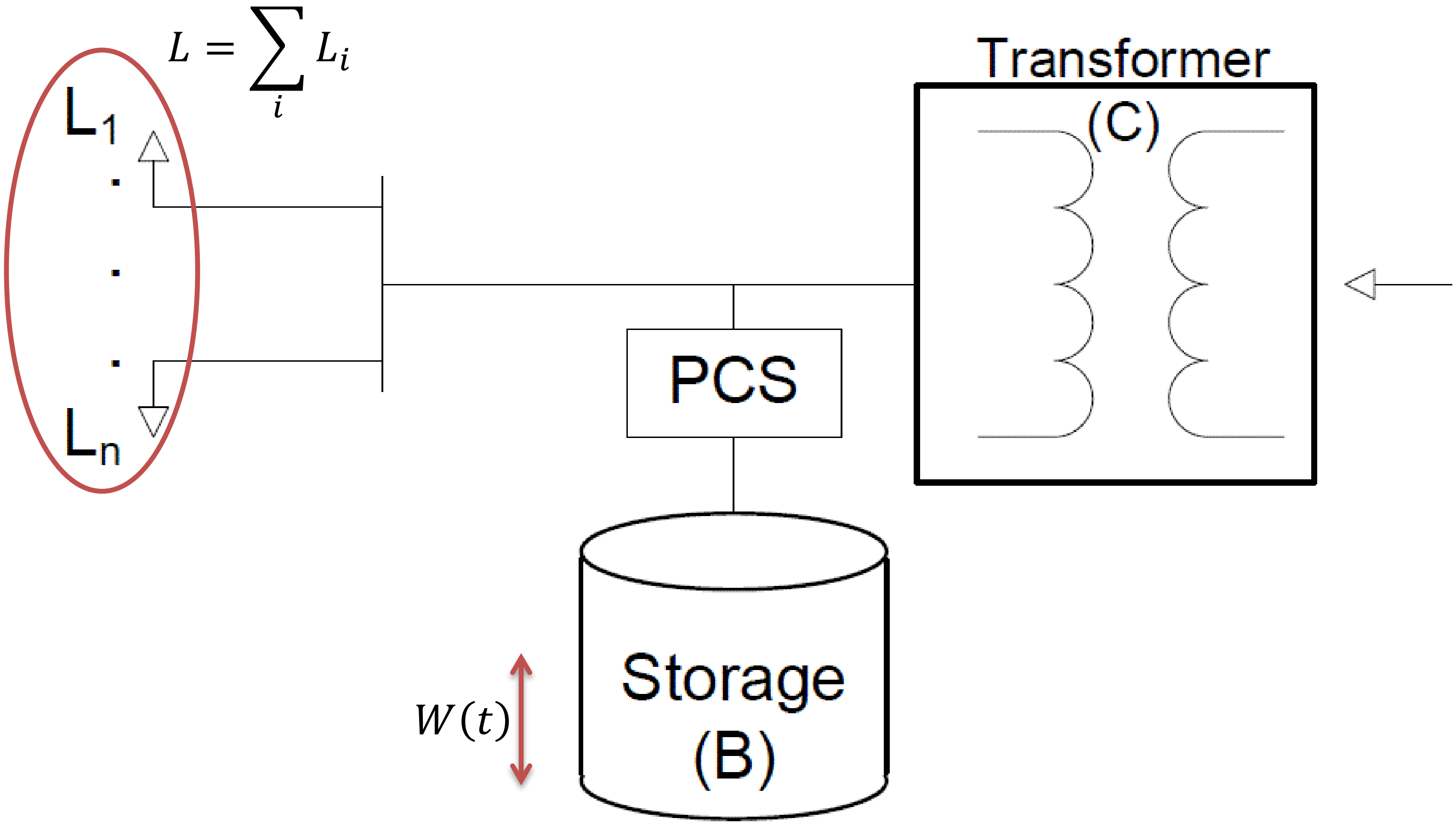
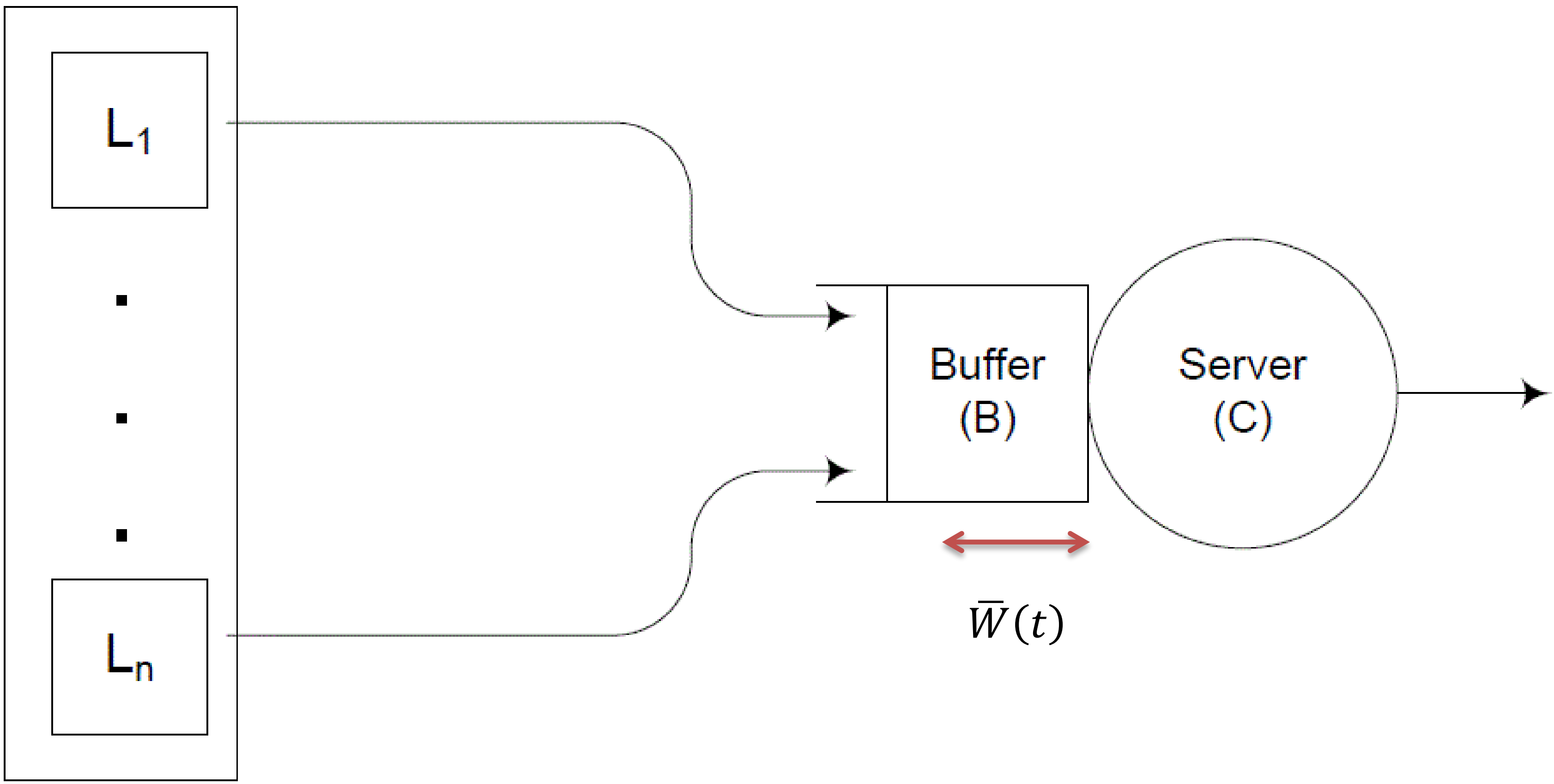
System Model:
Similar to a $D/G/1/B$ Queuing Model

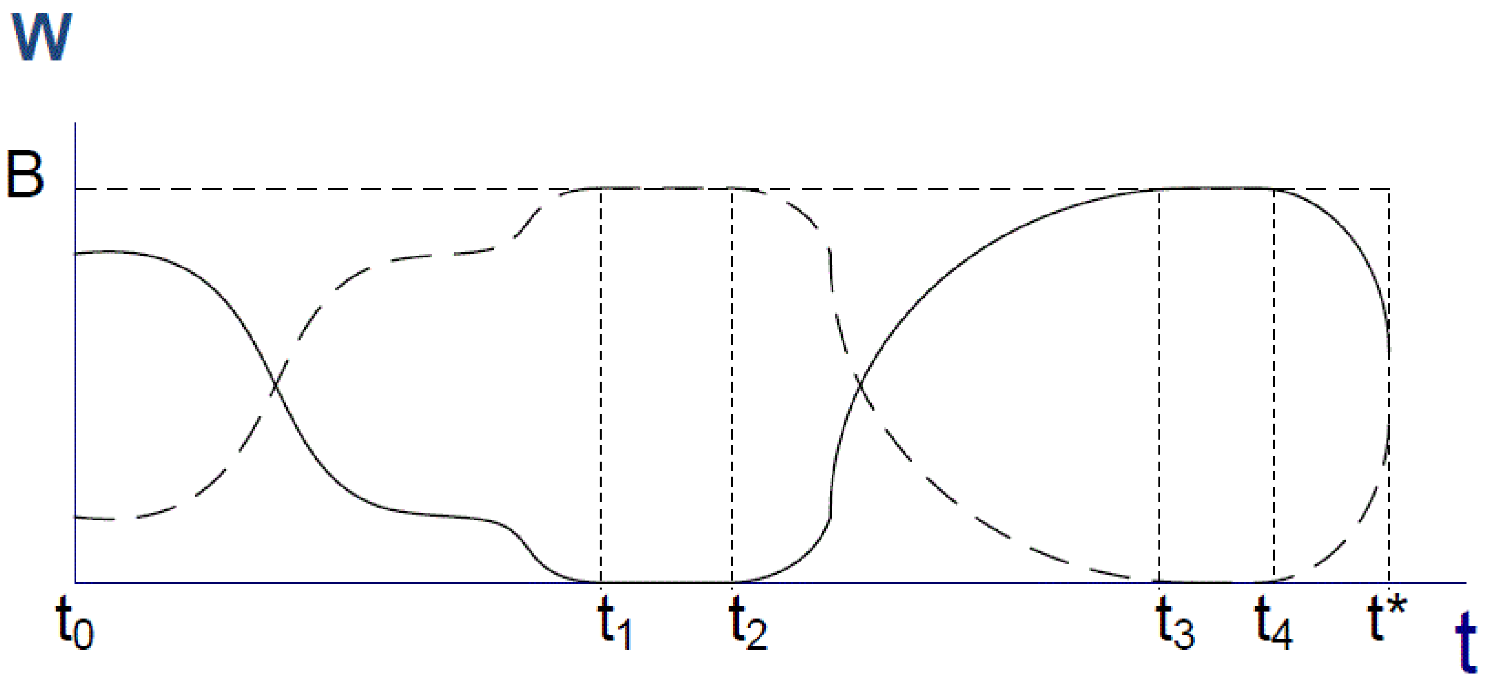


Equivalent Model:
A $G/D/1/B$ Queuing Model



Equivalence

- **Equivalence Theorem**: Every workload trajectory in the $D/G/1/B$ queuing system corresponds to an equivalent trajectory in the $G/D/1/B$ queuing system such that ∀t, $W(t)+\bar{W}(t) = \mathrm{B}$.



Equivalence

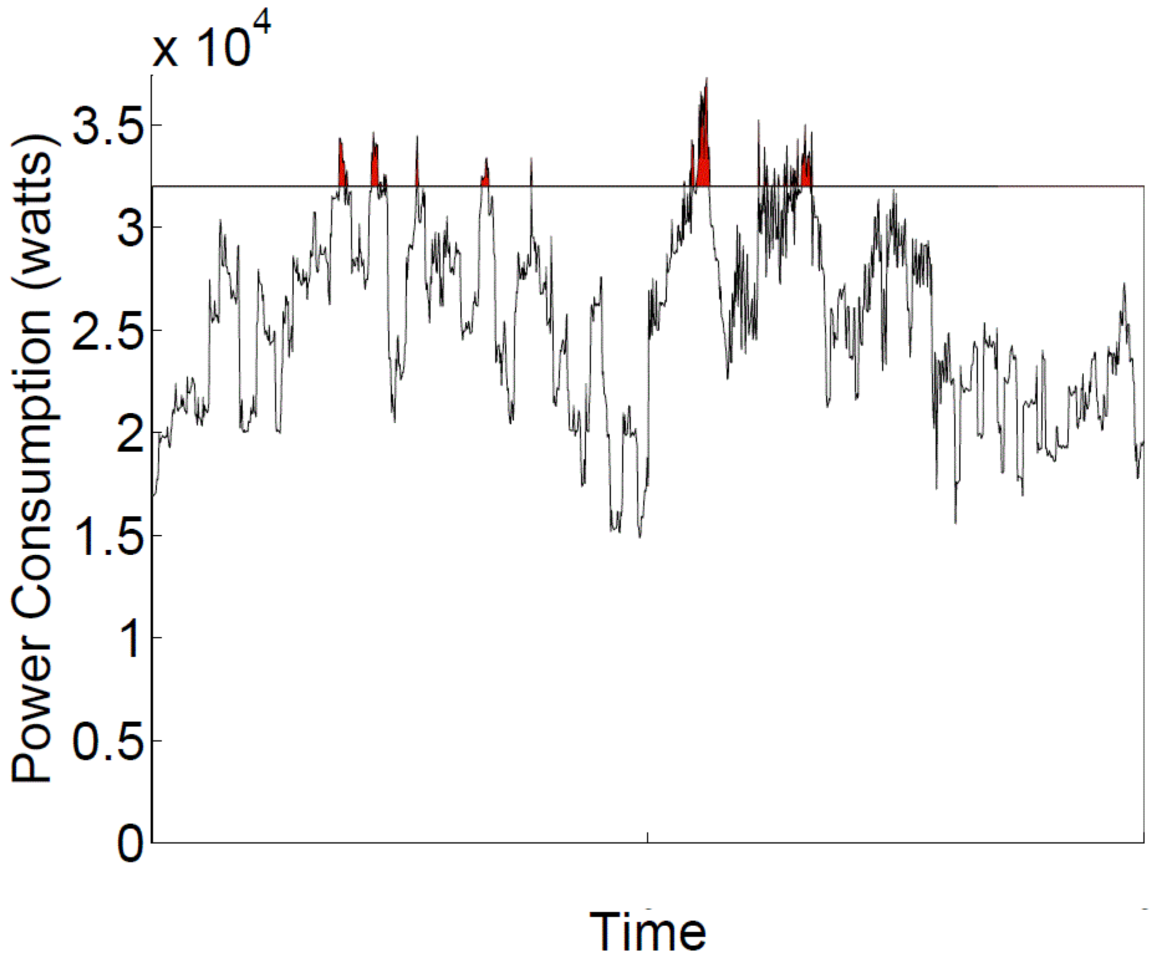
- **Corollary**: It follows from the equivalence theorem that:

$$\mathbb{P}(W(\infty) > B) = \mathbb{P}(\bar{W}(\infty) < 0)$$
$$\mathbb{P}(W(\infty) < 0) = \mathbb{P}(\bar{W}(\infty) > B)$$

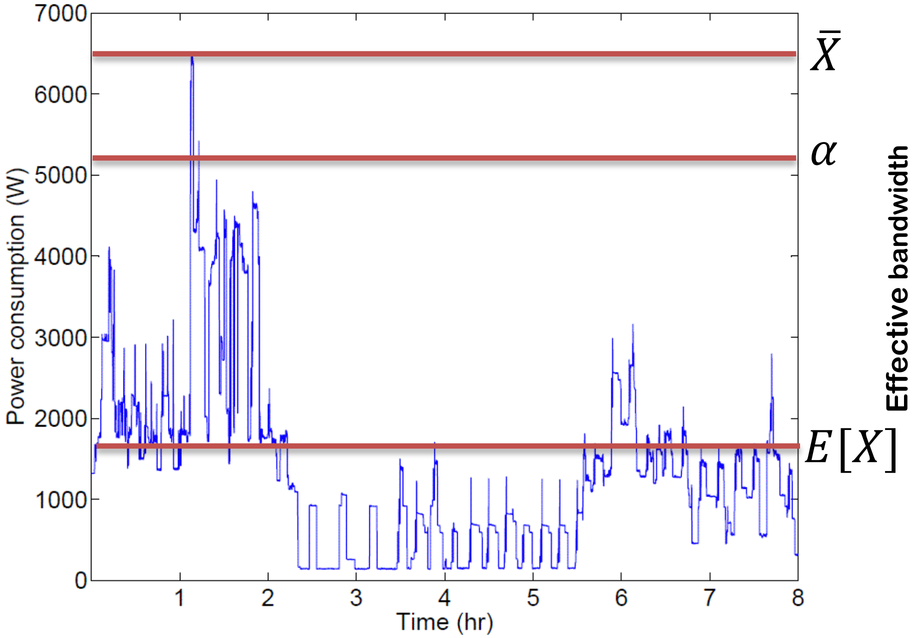
Where $W(\infty)$ is the stationary workload process.

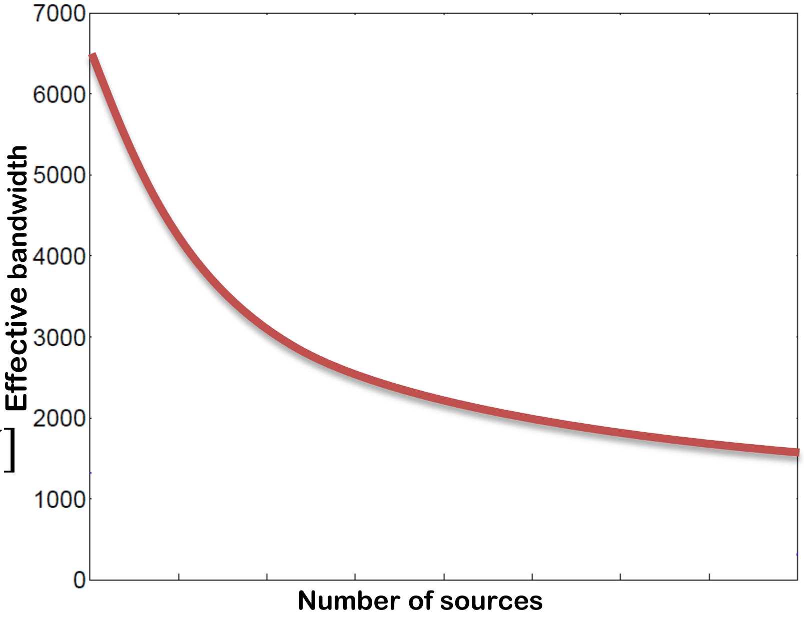
Loss probability ≡ Buffer overflow probability ≡ Storage underflow probability

Sizing the Grid



Effective Bandwidth





Effective Bandwidth

- It is defined as:

$$\alpha(s) = \frac{1}{s}\log E[e^{sX}] = \frac{1}{s}\log M(s)$$

- Additive property of effective bandwidth:

$$\alpha(s) = \sum_{i=1}^{I} n_i \alpha_i(s)$$

- Moment generating function of a Markovian source (defined by $\langle P, R \rangle$):

$$M(s) = \sum_{i} \pi_i e^{s r_i}$$

Capacity Region

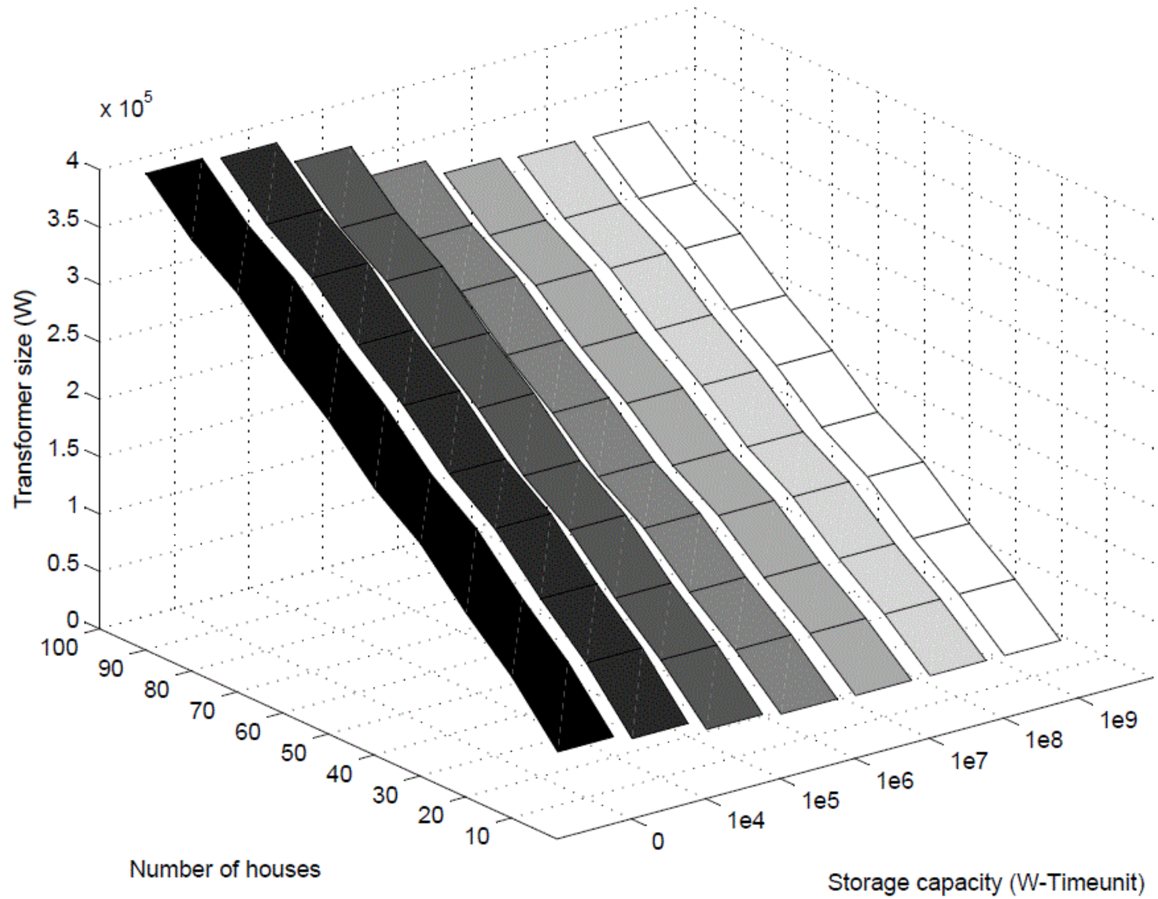
- Bufferless:

$$P(Y \geq C) \leq \epsilon \Longrightarrow Capacity\ Region$$
$$= \{C|inf_s\{s(\alpha(s) - C)\} \leq \log \epsilon\}$$

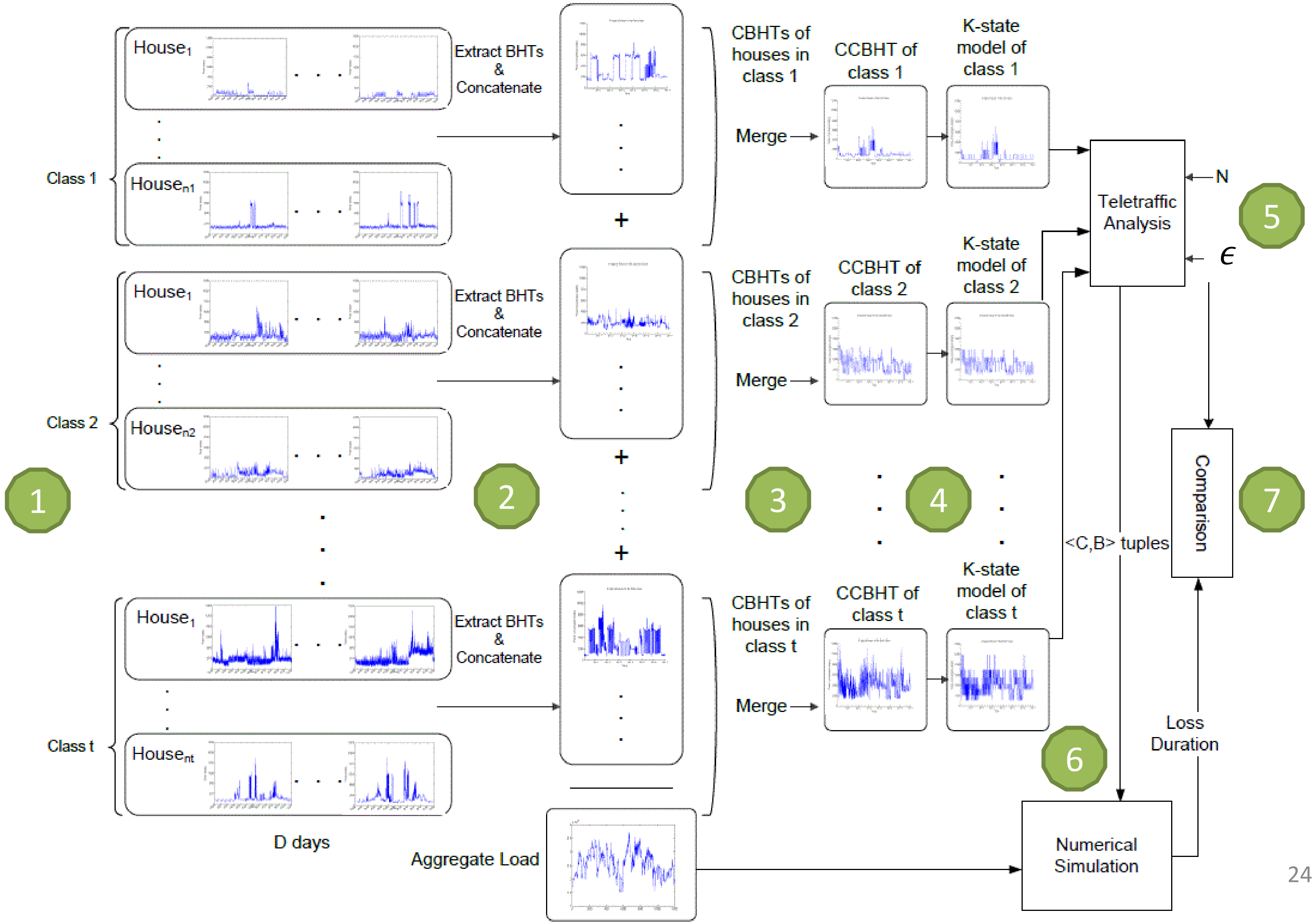
- Buffered:

$$P(W(\infty) \geq B) \leq \epsilon \Longrightarrow Capacity\ Region$$
$$= \{(C,B)|inf_s\{s(\alpha(s) - C)\} + zB \leq \log \epsilon\}$$

Capacity Region



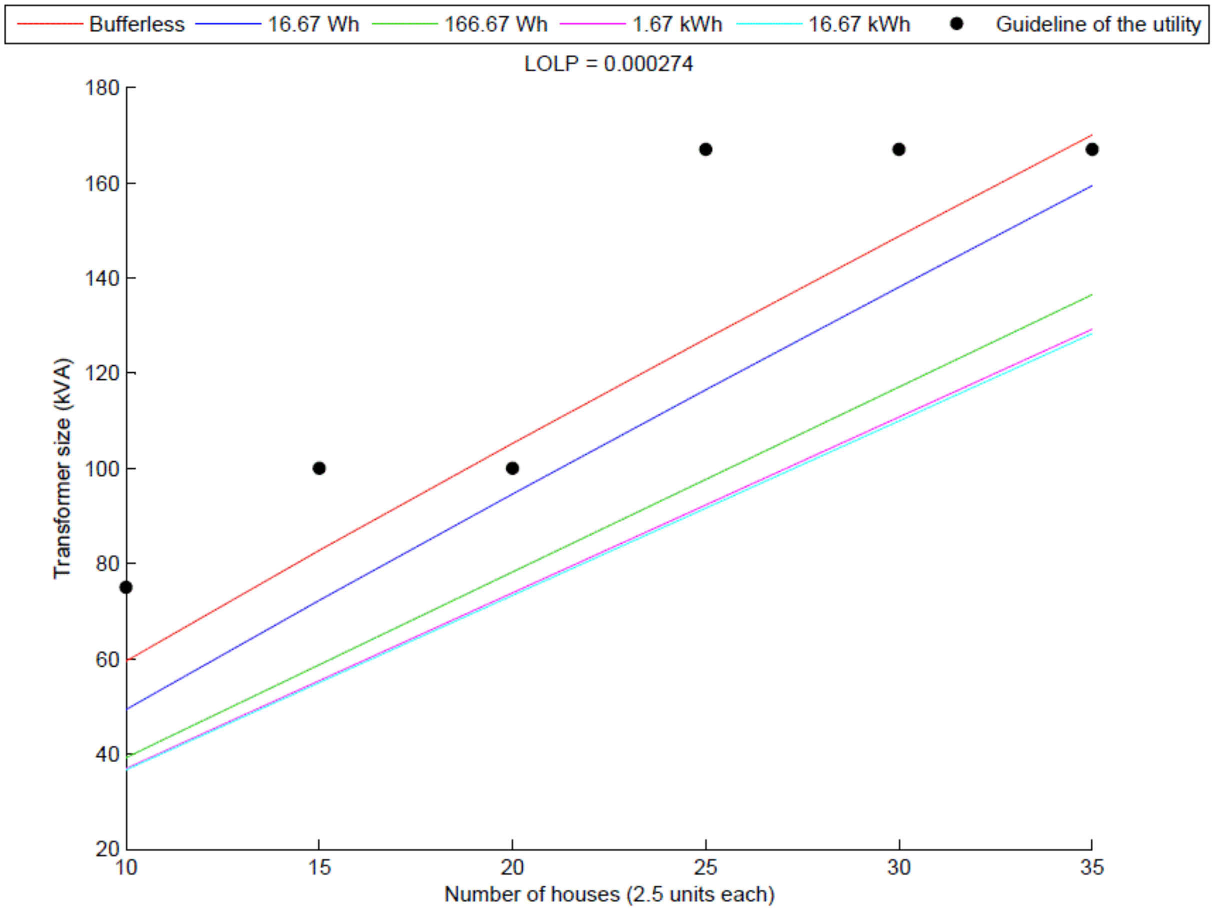
Teletraffic-based Sizing and Validation



Validation

- Comparing our sizing with industry practice
- Comparing results from numerical simulation and teletraffic theory
  - Data is obtained from our testbed
  - Data is obtained from the load simulator developed at University of Loughborough

Validation: Comparison with Industry Practice



Conclusion

- We construct Markovian reference models for the home loads of each class
- We prove that a branch of the electrical gird is equivalent to a queuing system
- We present an approach to jointly size a transformer and a storage system
- We show that our transformer sizing approach is consistent with industry practice

Thank you!

Any Questions?