Perfectly Orderable Graphs

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Perfectly Orderable Graphs

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- **background**
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graph colouring

- **assigning colours to vertices so that adjacent vertices get different colours**
- $\chi(G)$ **chromatic number**
- $\chi(G) \leq k$**?** **NP-complete**

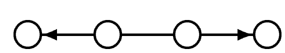
a simple algorithm

- **order vertices, label colours**
- **greedy-colour:**
  in order, for each vertex,
  use smallest available colour

- **for which** G **is there a vertex order**
  for which greedy$(G) = \chi(G)$**?** **all** G

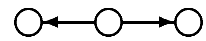
perfectly orderable graphs

- **[Chvátal 83] perfectly orderable iff there is a vertex order for which, for all induced subgraphs** H, **greedy**$(H) = \chi(H)$

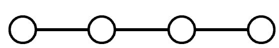
- **[Chvátal 83] perfectly orderable iff acyclic edge orientation with no** 

perfectly orderable graphs: examples

origin

- **triangulated:**
  acyclic edge orientation with no 

- **comparability:**
  acyclic edge orientation with no 

- 

- **(triangulated** $\cup$ **comparability)** $\subset$ **PO**

context: perfect graphs

- G **perfect:** $\chi(H) = \omega(H)$ **for all** $H \leq G$
- **not:** C_5, $C_7, \overline{C}_7$, $C_9, \overline{C}_9$, $C_{11}, \overline{C}_{11}$, ...
- **Berge graph: no** C_{2k+1} $\overline{C}_{2k+1}$ $_{k\geq 2}$
- **[Berge 60]:** G **perfect** $\stackrel{?}{\Longleftrightarrow}$ G **Berge**

- **[Lovász 72]:** G **perfect** $\Longleftrightarrow$ $\overline{G}$ **perfect**

- **recognition? (co-NP [Lubiw 84])**

- **[Groetschel/Lovász/Schrijver 84]: find** χ, ω **in polynomial time**

perfect graphs: current directions

- **Berge subclasses**
  - **weakly triangulated: no** $C_{k\geq 5}$ $\overline{C}_{k\geq 5}$
    - **perm'n, cographs, (co-)tri'd, (co-)chordal bip', . . .**
    - $O(n^4)$ **recognition/optimization**
    - **structure: handles, two-pairs**
  - **perfectly orderable**
  - **perfectly contractile, quasi-parity**
  - . . .
- P_4**-structure**
  - **self-complementary:**
  - **recognition?**
  - **perfectly orderable graphs**
- **perfection/imperfection properties**
- . . .

background

- **class relations**
  - **subclasses:**
    cographs, (co-)tri'd, comparability
  - **superclasses:**
    perfectly contractile, perfect
  - **forbidden** C_j $\overline{C}_j$**:**
    $C_{2k+1 \ \ k \geq 2} \quad \overline{C}_{j \ \ j \geq 5}$

- **[Hammer/Mahadev 85]**
  bithreshold graphs

- **[Chvátal/Hoàng/Mahadev/deWerra 87]**
  four classes

- **[Hoàng/Khouzam 88]**
  brittle graphs

- **[Chvàtal 89]**
  co-chordal bipartite graphs

- **[Middendorf/Pfeiffer 90]**
  recognition NP-complete

co-chordal bipartite graphs are PO

- **chordal bipartite:**
  bipartite and no $C_{2k} \quad {}_{2k \geq 6}$
- **doubly lexical matrix order:**
  row/column vectors lexically increasing
- **totally balanced 0-1 matrix:**
  bipartite graph is chordal bipartite
- Γ**: submatrix** $\begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix}$
- **[Anstee/Farber 84 Lubiw 87**
  Hoffman/Sakarovich/Kolen 85]
  totally balanced iff
  every doubly lexical order is Γ**-free**
- **co-chordal bipartite graphs are PO**

two conjectures

- **[Chv 89] graphs with**
  no $C_{2k+1} \quad {}_{k \geq 2}$
  no $\overline{C}_j \quad {}_{j \geq 5}$
  no P_5
  are perfectly orderable

- **[Chv 89] graphs with**
  no $C_{2k+1} \quad {}_{k \geq 2}$
  no $\overline{C}_j \quad {}_{j \geq 5}$
  no bull
  are perfectly orderable

two new results

- **[Hayward 95] graphs with**
  no $C_{2k+1} \quad {}_{k \geq 2}$
  no $\overline{C}_j \quad {}_{j \geq 5}$
  no P_5
  are perfectly orderable

- **[deFigueiredo/Maffray/Porto 94 Hayward 98] graphs with**
  no $C_{2k+1} \quad {}_{k \geq 2}$
  no $\overline{C}_j \quad {}_{j \geq 5}$
  no bull
  are perfectly orderable

proving theorem 2: reduction

- **[dFMP 94] box partition**

- **[dFMP 94] reduction: iff graphs with**
  no $C_j \quad {}_{j \geq 5}$
  no $\overline{C}_j \quad {}_{j \geq 5}$
  no bull
  no P_6
  co-box partition
  are perfectly orderable

  (contains co-chordal bipartite graphs)

proving theorem 2: generalization

- [Chv 89] for every co-bipartition of a co-chordal bipartite graph, every co-DLO extends to a PO

- [H 95] for every ***** co-box partition of a ******* ******* graph,
  - every co-DLO extends to a PO

    iff no 8-fig

. . . and conclusion

– [H 98] some co-DLO induces no 6-fig

proving theorem 2: the computer