Hex and
theoretical computing science:
60 years and beyond

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Thankyou:

Natural Sciences and Engineering
Research Council of Canada

University of Alberta
Games Research

- theoretical computing science
  - some highlights

- Hex and theoretical computing science
  - 1942 to present

- Hex and theoretical computing science
  - the future ?

- **4 colour problem**
    - **1852 Guthrie**   **4 colours suffice?**
    - **1879 Kempe**   **"proof"**
    - **1890 Heawood**   **not!**
    - **1880 Tait**   **"proof"**
    - **1891 Petersen**   **not!**
    - **...**
- **1900 Hilbert**   **23 problems**
    - **goal: axiomatize/formalize mathematics**
- **1917 Hilbert   every math question decidable**
- **1931 Gödel   incompleteness**
- **1936 Turing   incomputability**

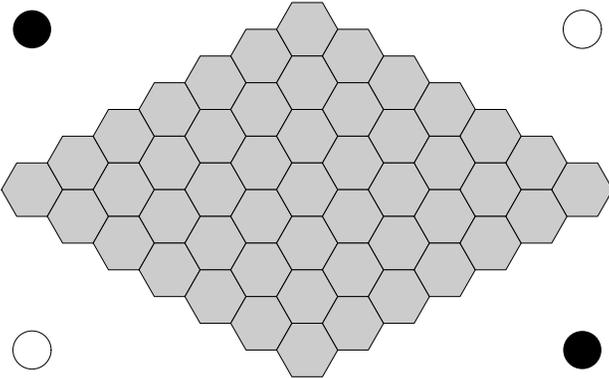- mind? intelligence? computation?
- 1936 Turing
  - model of computation   Turing machine
    * deterministic
    * universal (programmable)
  - halting problem incomputable

theoretical computing science   to complexity

- 1947 Dantzig   simplex
- 1940s Shannon circuit/information complexity
- 1960 Berge   perfect graphs
- 1965 Edmonds   matching ("good" algorithm)
- 1971 Cook/Levin/Karp   NP-completeness
  - P   solvable in poly'l time
  - NP   yes-verifiable in poly'l time
  - PSpace   solvable in poly'l space
  - $P \subseteq NP \subseteq PSpace$
- $P = NP$?
- 2000 Clay   \$1 000 000 prize

- stronger models of computation

  – randomization?

  – quantum computing?

- approximation

  – 1991 Feige/Goldwasser/Lovasz/Safra/Szegedy
    PCP theorem: approximation can be hard

- polynomial not always practical

  – 1979 Khachian   linear programming

  – 1983-95 Robertson/Seymour
         minor-closed graph class recognition

- heuristics

  – Turing 1950   "can machines think?"

- computers and proof: 4 colour problem

  – 1976 Appel/Haken

  – 1996 Robertson/Sanders/Seymour/Thomas

1942 Hein

game criteria + 4-colour problem $\implies$ Hex
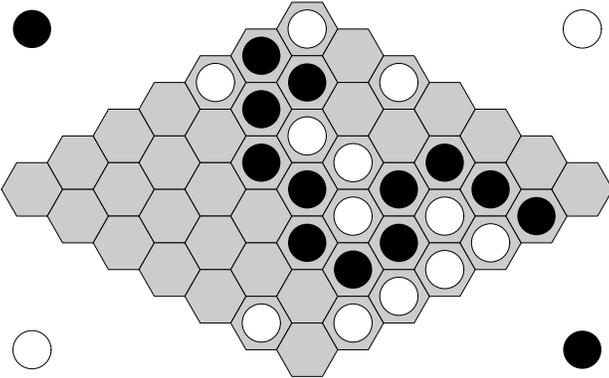
1948 Nash (& Gale)

"connecting topology and game theory"
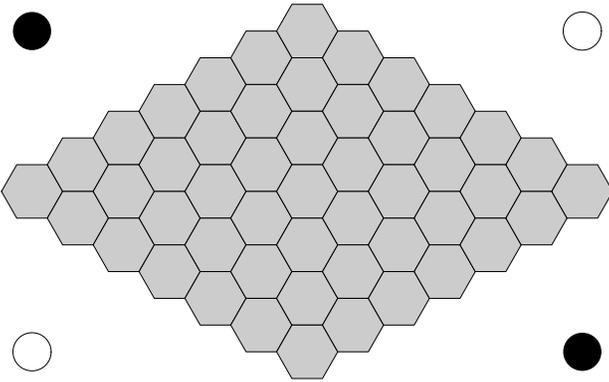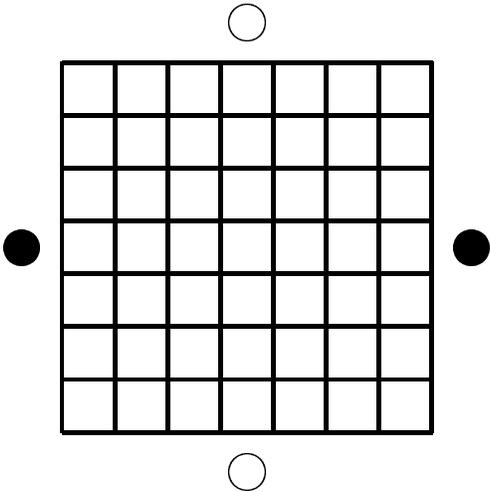
1953 Shannon (& Moore)

"the problem of designing game-playing machines is fascinating"

paradoxically, the positional judgment of this machine was good; its chief weakness was in end-game combinatorial play"

1957 Gardner

"played on the tiles of the bathroom floor"
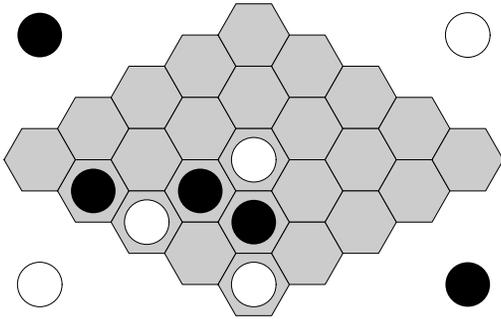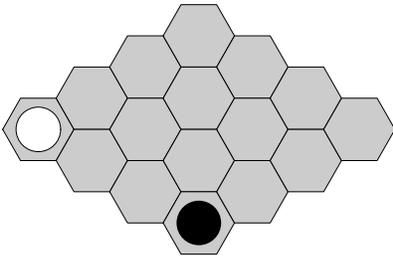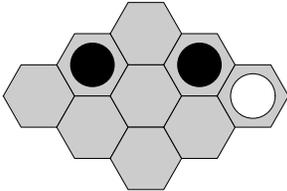
1975 Schensted & Titus

"whenever you feel you must use a car, try playing Y until the feeling passes"
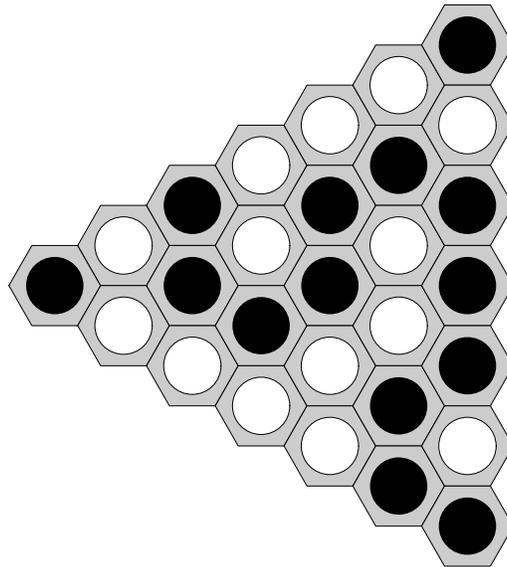
Hex special case of Y
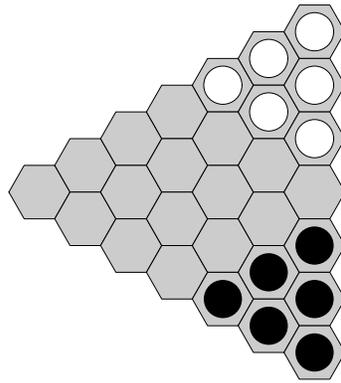
Schensted's Y-reduction
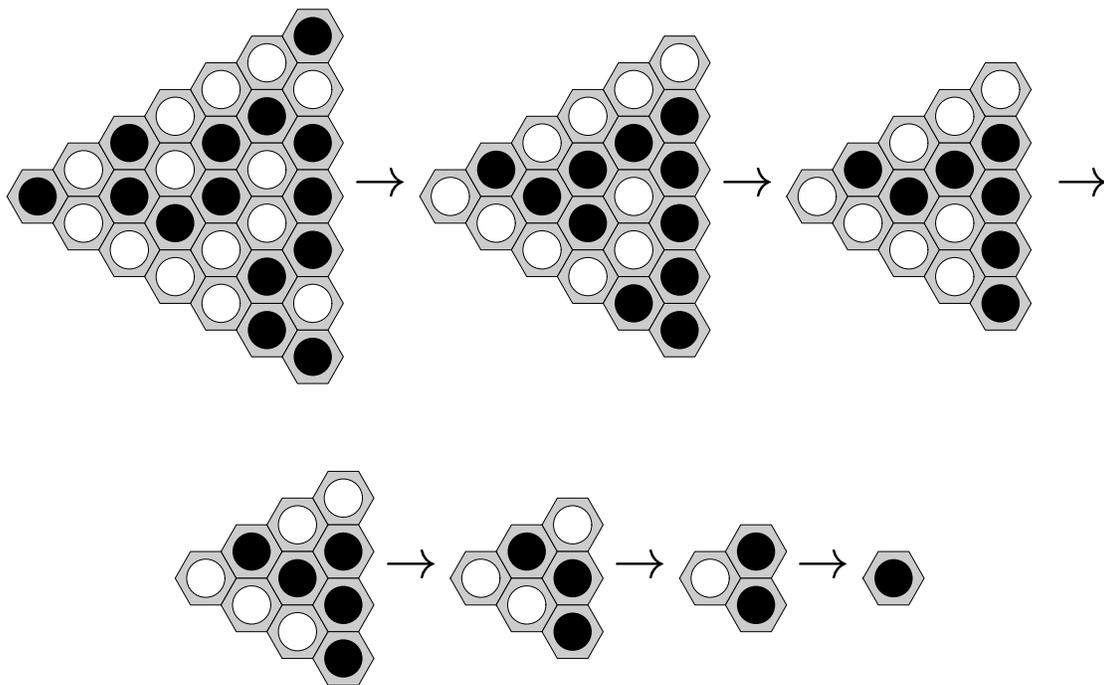
reversing Y-reduction
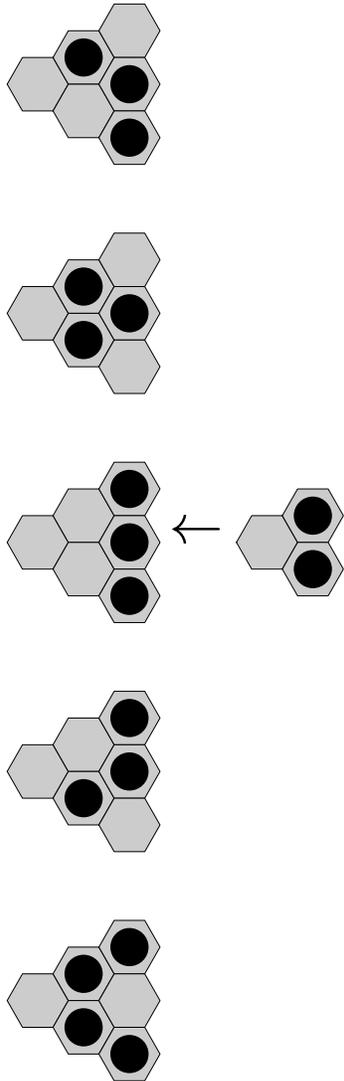
1977 Berge

"l'art subtil du Hex"
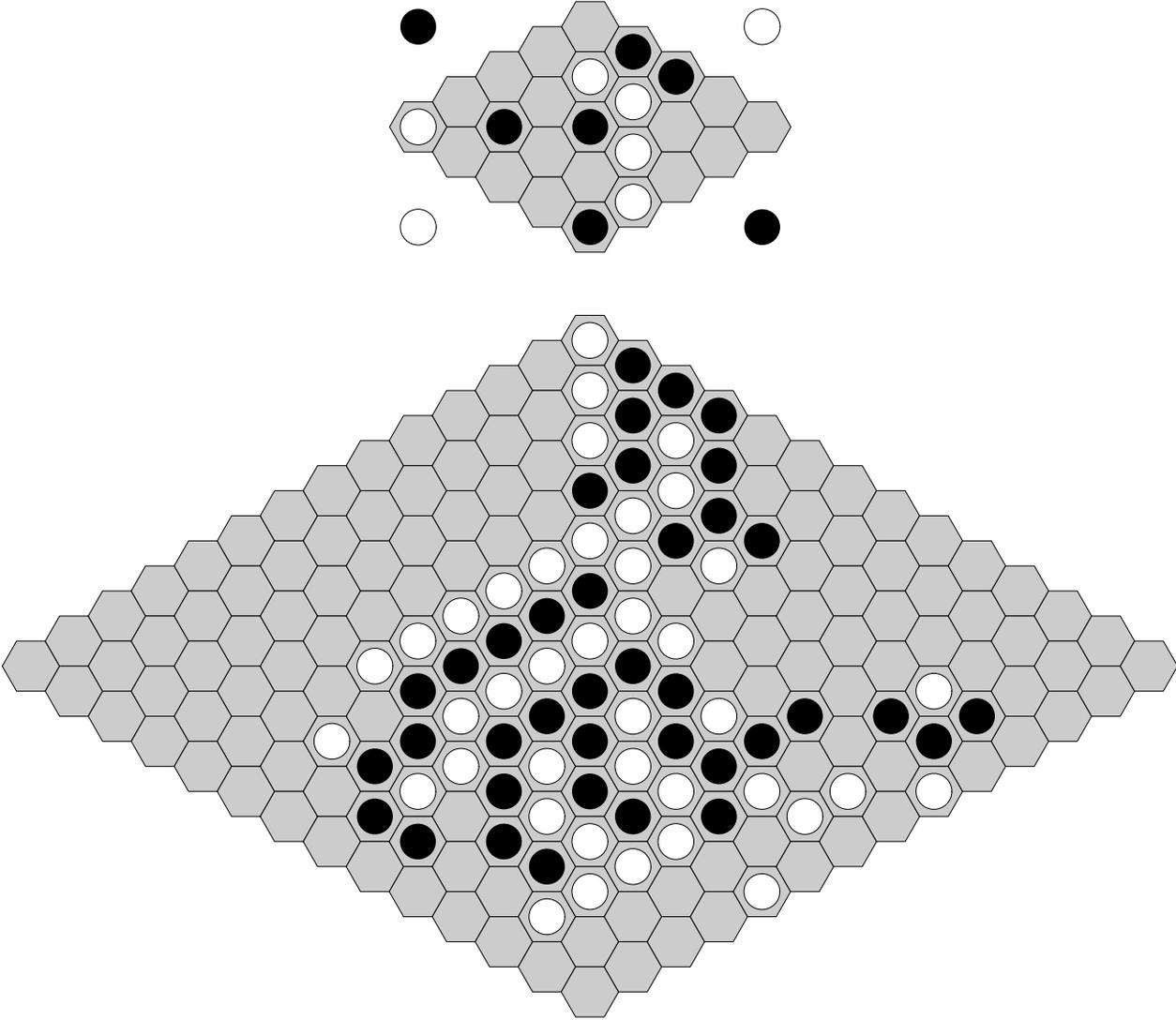
virtual connections, mustplay
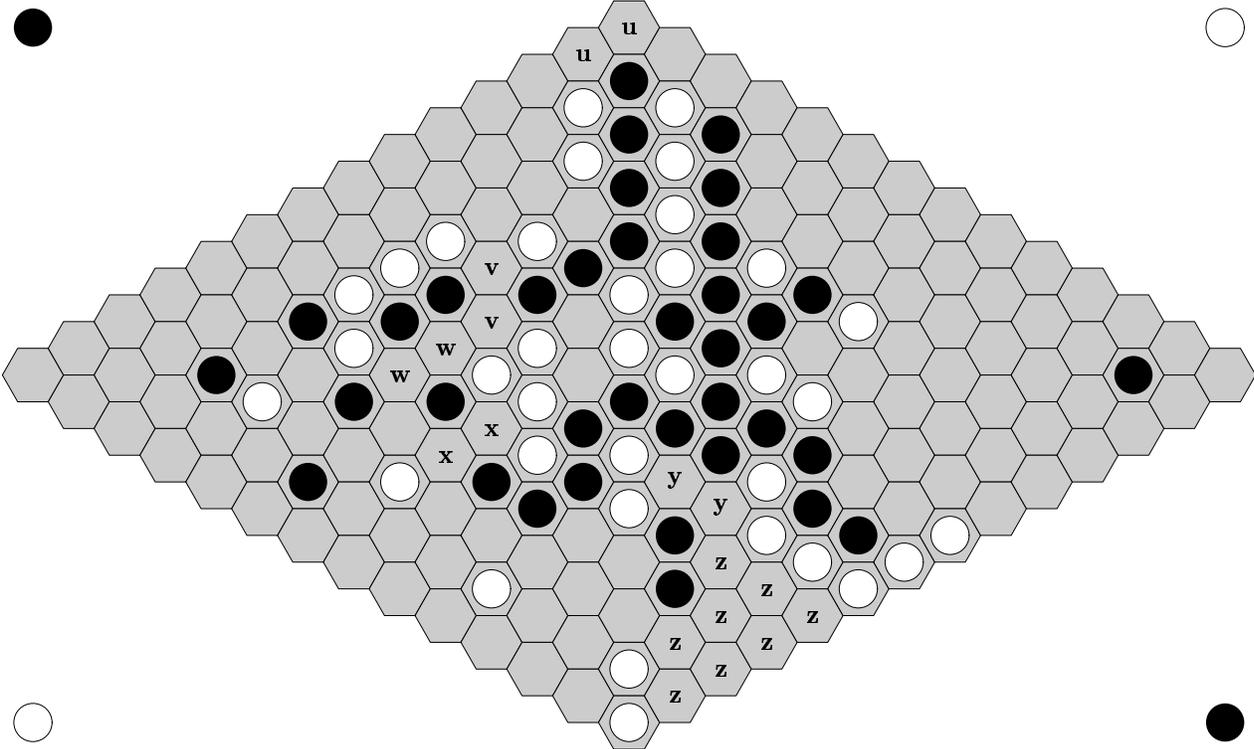
1981 Berge

"...to solve some Hex problem
  by using nontrivial theorems about
  combinatorial properties of sets ..."
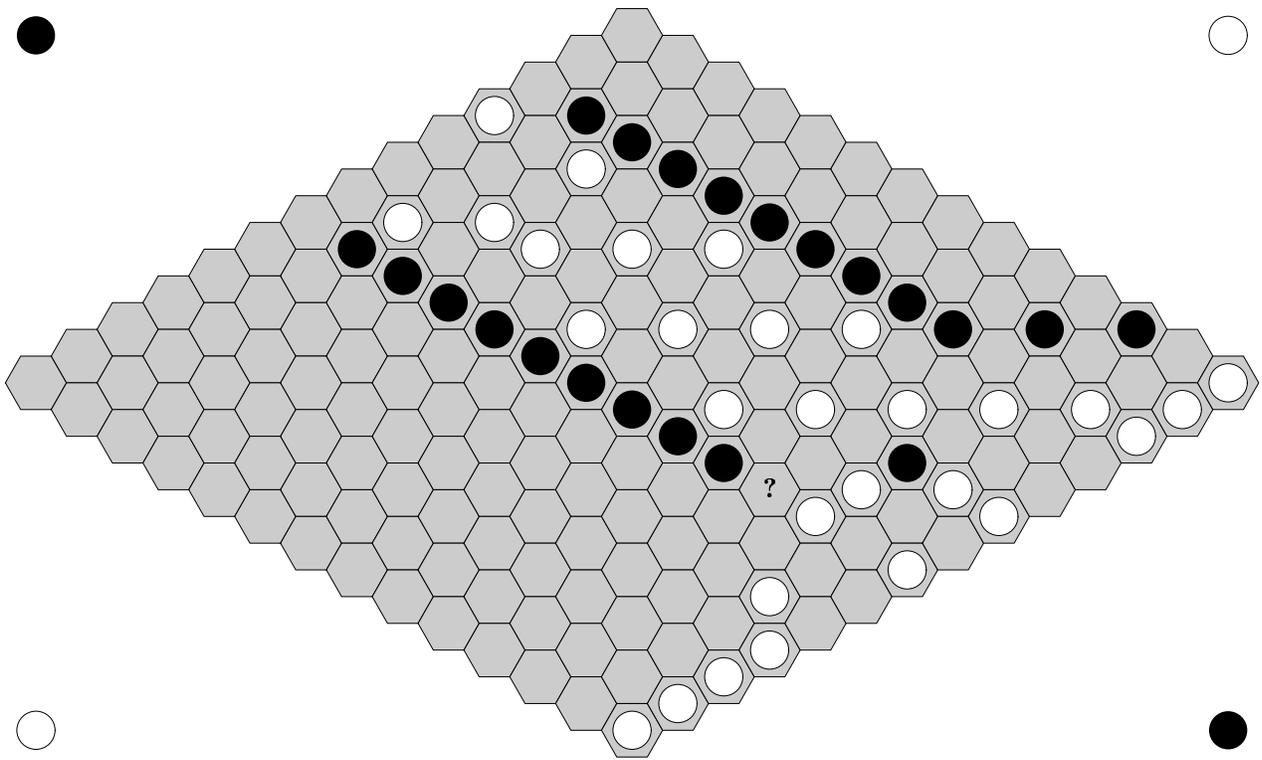
1976 Even & Tarjan

      generalization of Hex PSACE-complete

1981 Reisch

      Hex PSPACE-complete

1984 Berge

"computers will never beat humans at Hex"

Hex computer programs   Mongoose 2003/4

H,Yngvi Björnsson, Mike Johanson, Maryia Kazeke-
vich, Morgan Kan, Nathan Po, Jack van Rijswijck

- resistance network

- virtual connections

- dead cell analysis
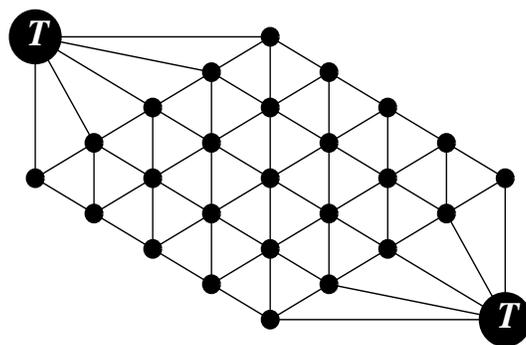
- mustplay
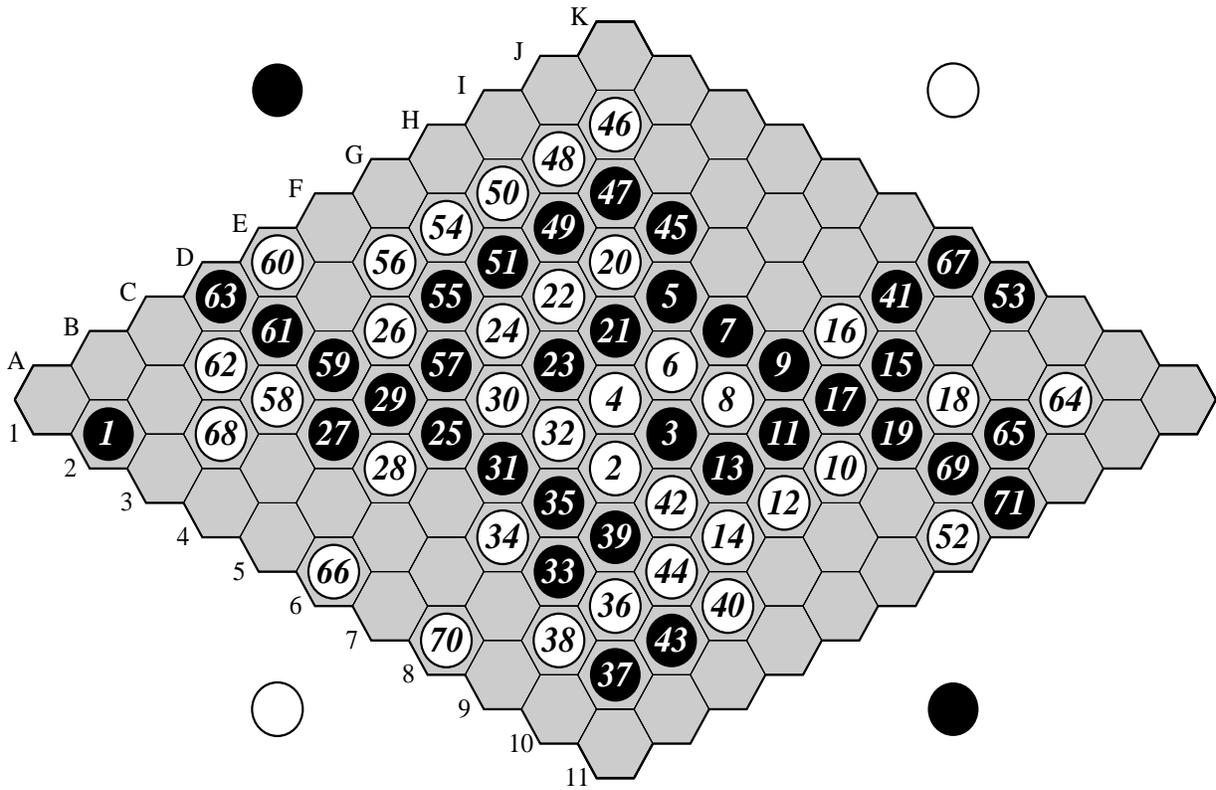
CGO 2004 Game 3. Six (black) defeats Mongoose.
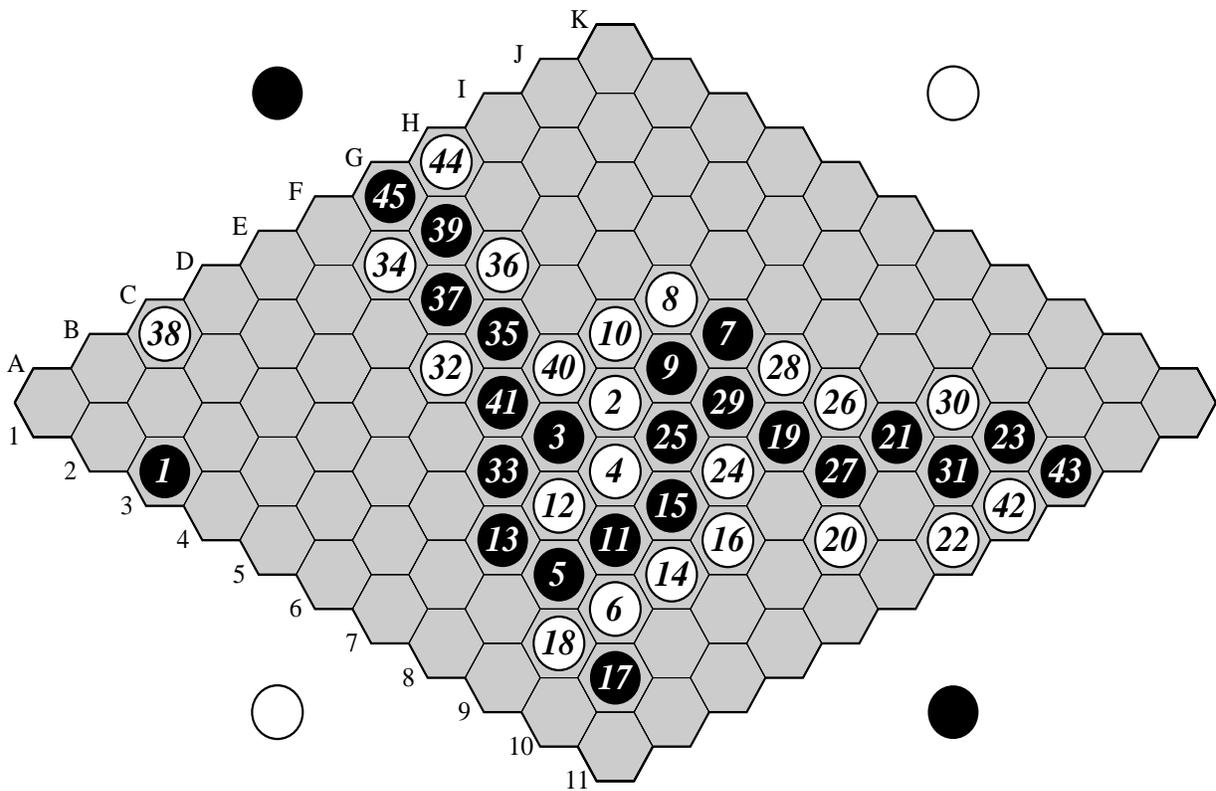
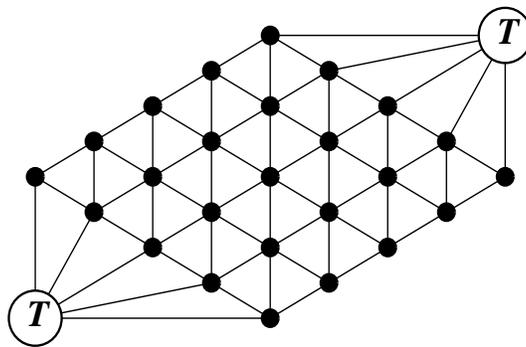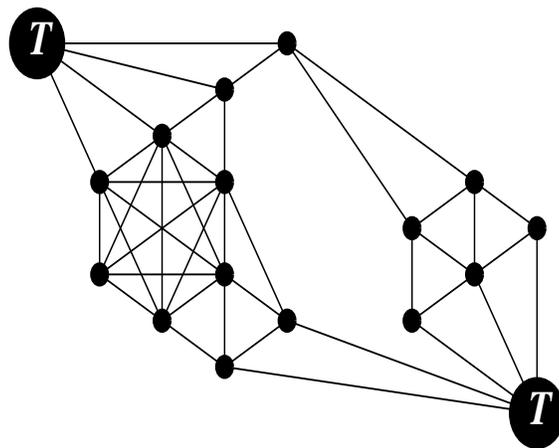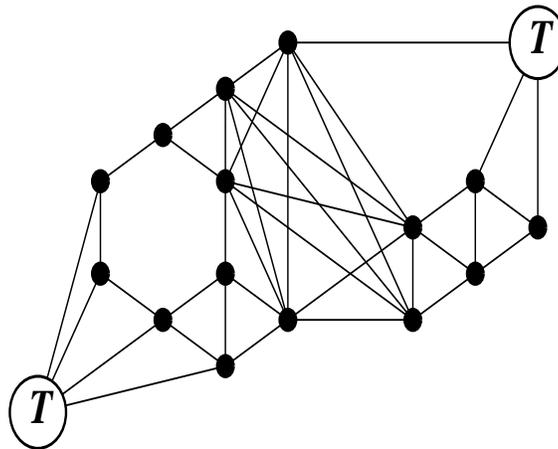**Game 4. Mongoose (black) defeats Six.**
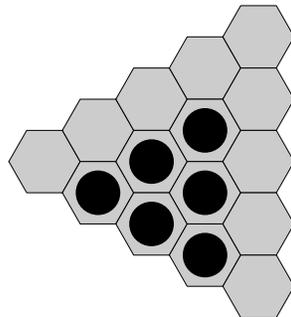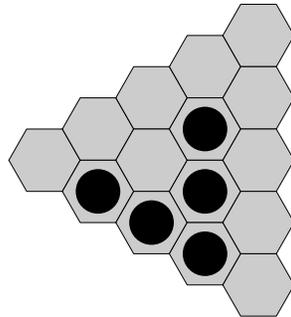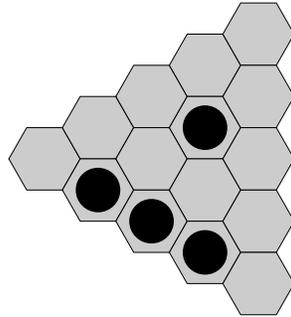
monophonic intervals
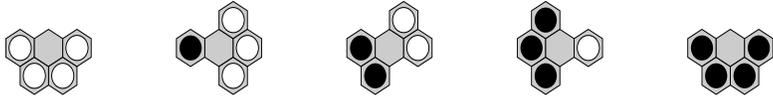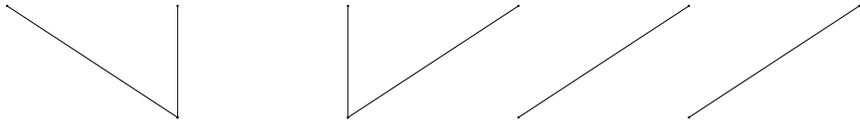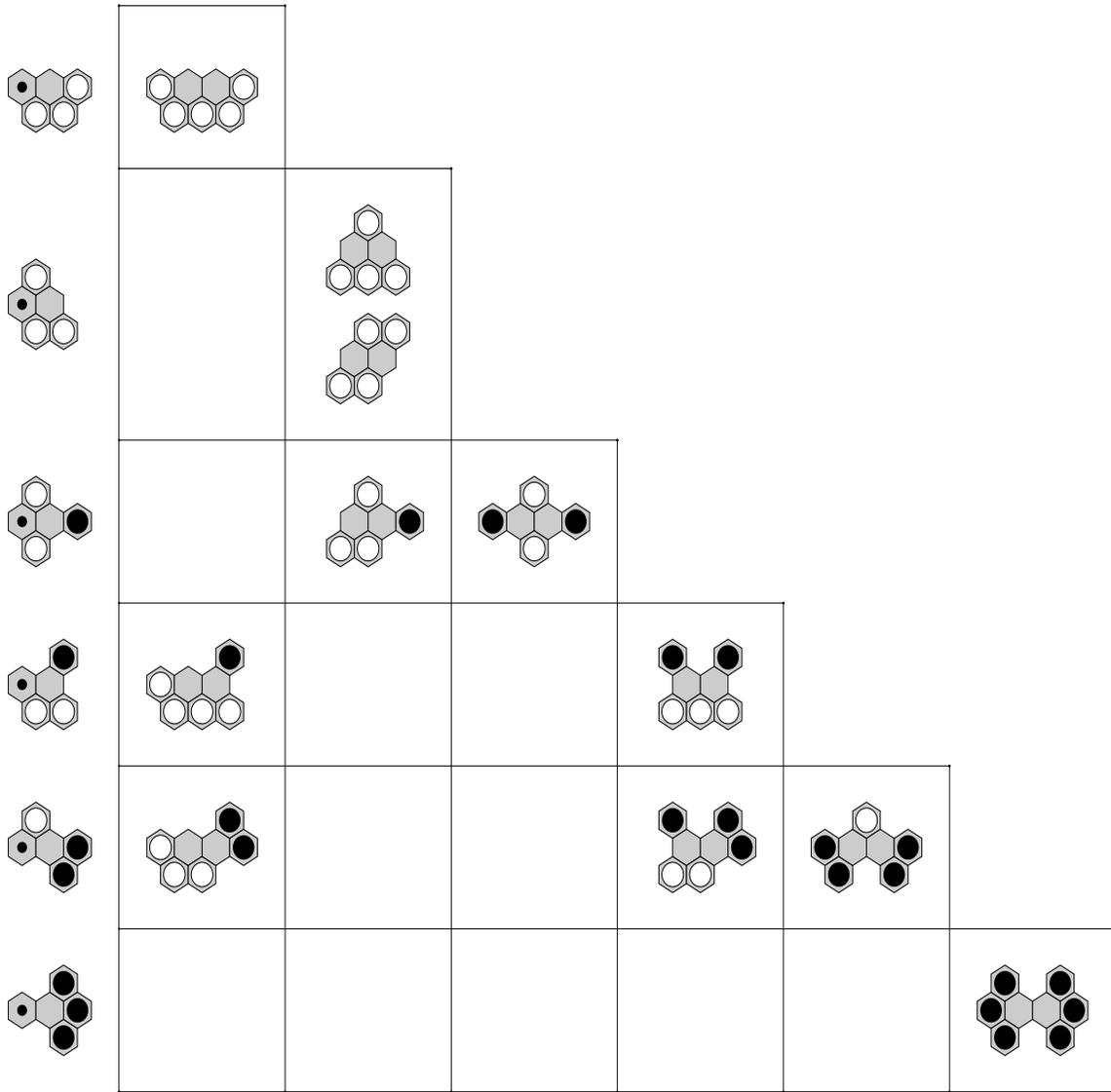
- node v is *dead* if,
  for every completion of $G - v$,
  colour of v does not change winner

- *live* iff not dead

- Theorem: live iff on terminal-terminal mono-phonic interval of reduced graph
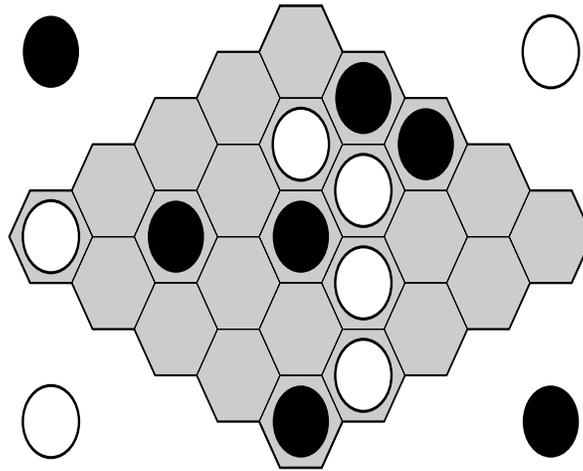
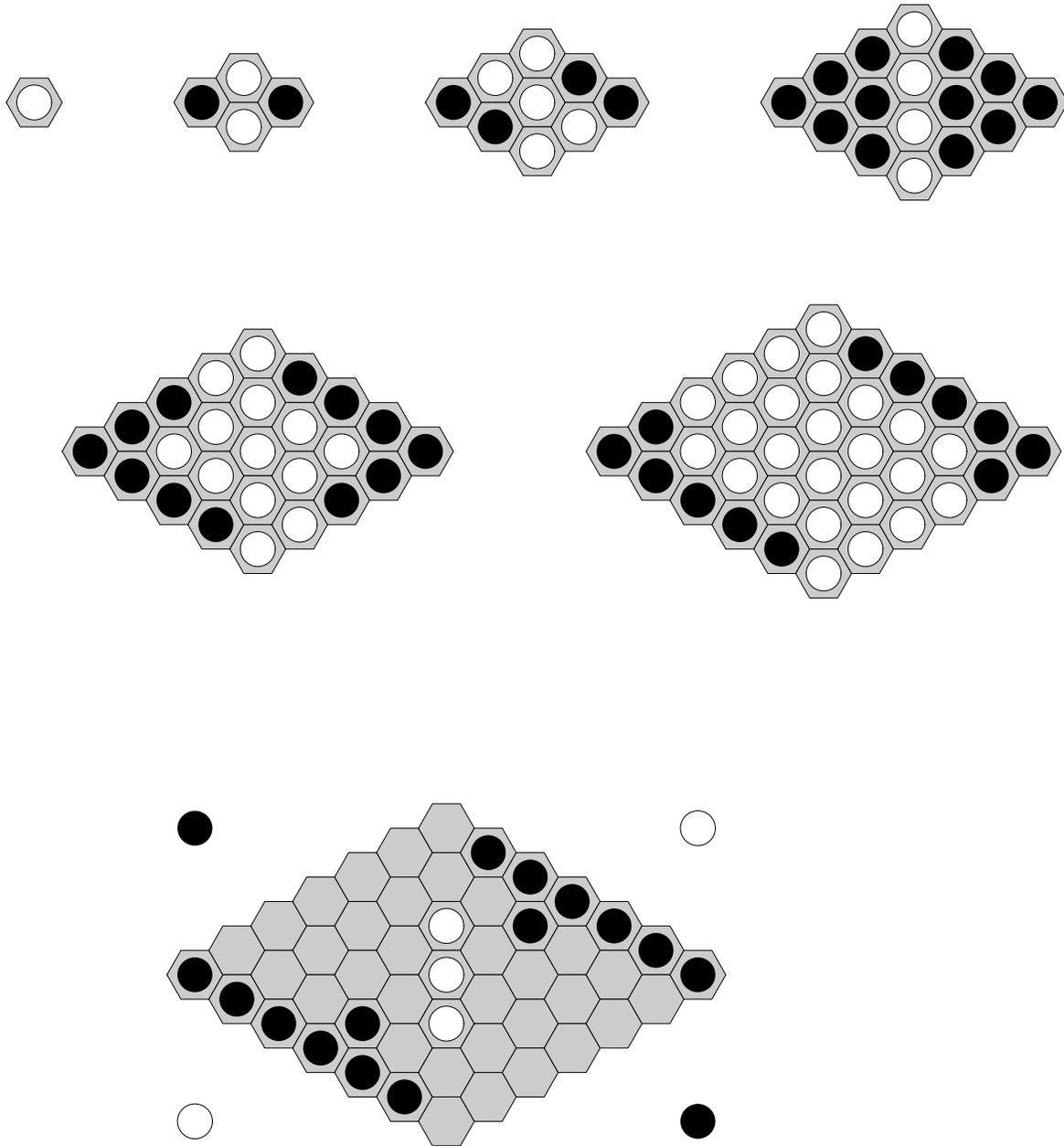- Fellows (Robertson/Seymour) computing monophonic interval NP-hard

Hex: what's next?

- computer player
  - machine learning games techniques
  - smarter machine learning?
  - better heuristics
- computer solver
  - PSpace verification techniques

solving Hex: opening winning moves

2002 H/Björnsson/Johanson/Kan/Po/Van Rijswijck