SOLVING 10x10 HEX

TCGA WORKSHOP KEYNOTE PRESENTATION

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computing UAlberta

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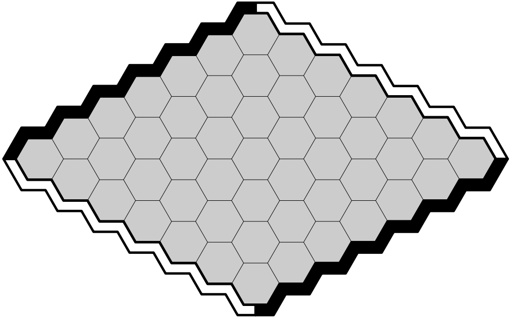
THANK YOU

- invitation Prof Mark Kayll UMontana
- solving 10×10 Hex joint with Jakub Pawlewicz
- builds on work with B Arneson, P Henderson
- machine Martin Müller
- photo courtesy MIT Museum, MIT, Cambridge MA
- Natural Sciences and Engineering Research Council of Canada

1942 HEX

RULES

- 2 players, alternate moves
- win: connect your two sides



1942 HEX

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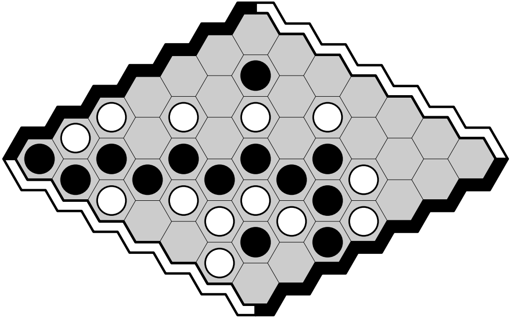
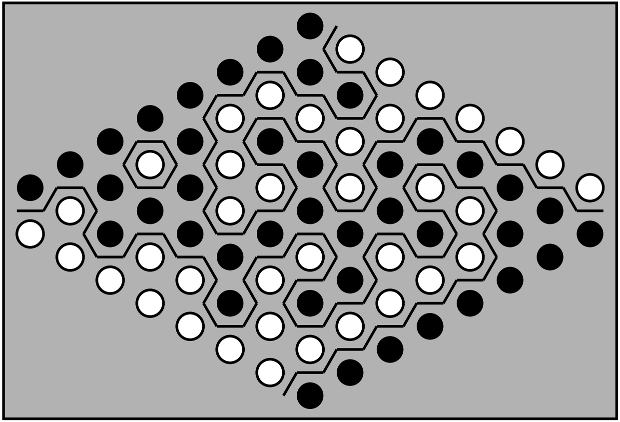


N X N HEX: 1ST-PLAYER WIN

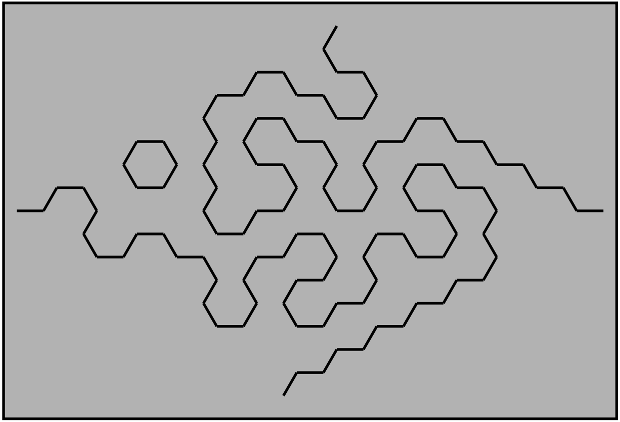
PROOF

- lemma: extra X-cell ok for player X
- lemma: no draws
- suppose P2 has win strategy S2
- then P1 can move anywhere, forget move, and follow S2
- thus P1 has win strategy, contradiction □

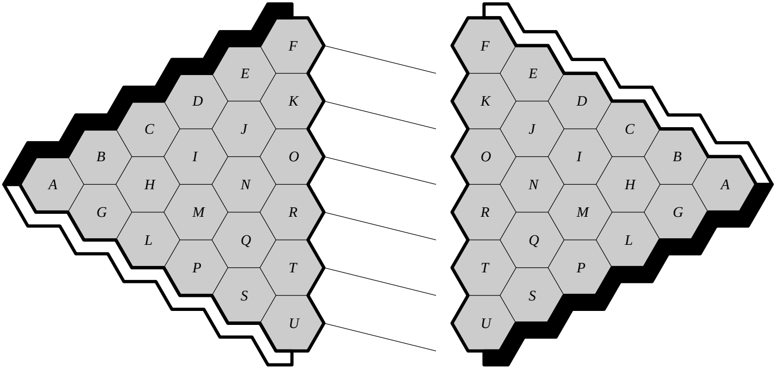
NO-DRAW



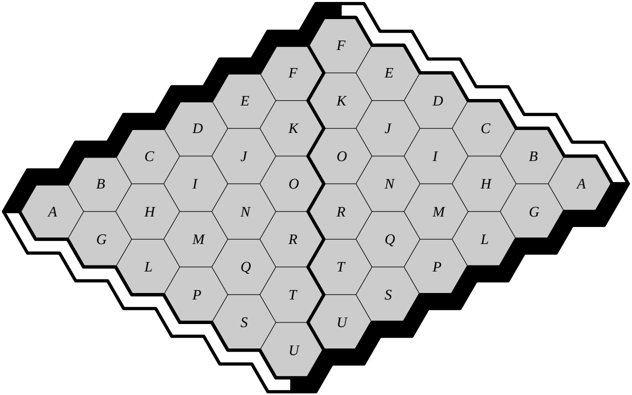
NO-DRAW



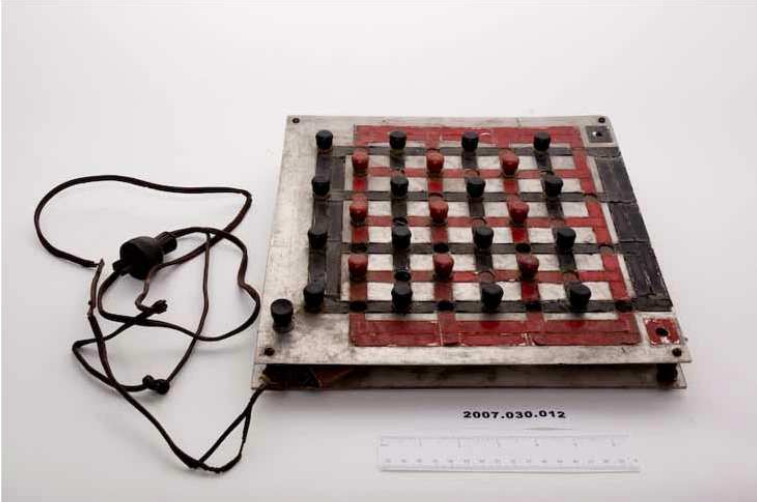
N X N+1 HEX: LONGER-SIDE WIN



N X N+1 HEX: LONGER-SIDE WIN



1951 Shannon machine

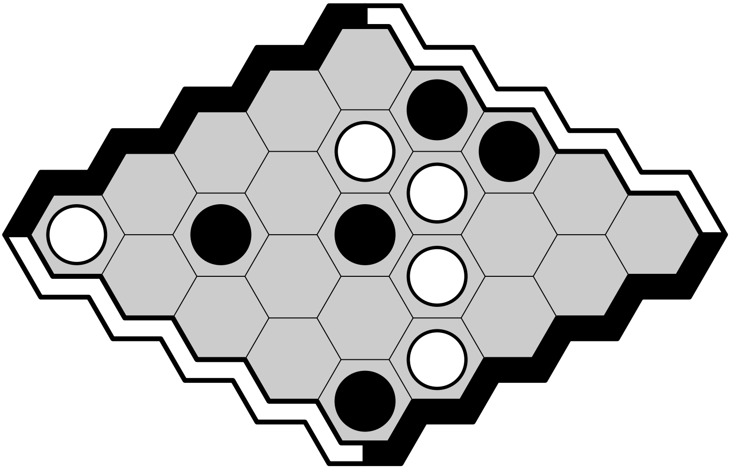


1951 Shannon machine

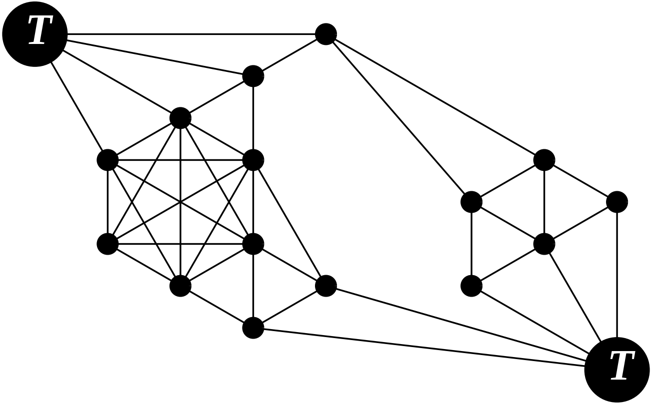
- play on any graph
- two marked vertices
- black move: 'short' any vertex (make nbrs clique)
- white move: 'cut' any vertex (delete)
- black wins iff two marked vertices are shorted (connected)

- generalizes Hex

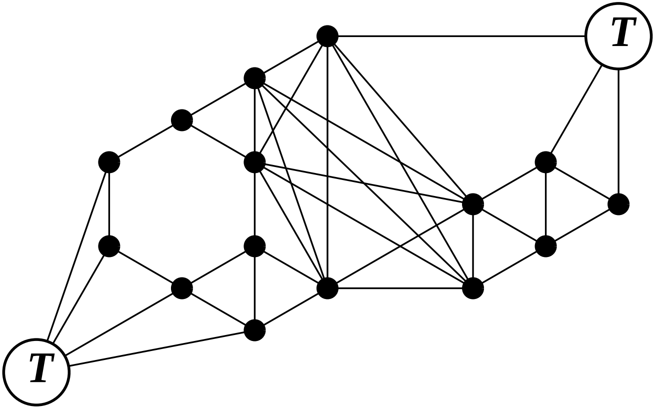
1951 Shannon machine



1951 Shannon machine



1951 Shannon machine



PROVABLY HARD

• 1975 Even & Tarjan	Shannon v-switching: PS-c
• 1981 Stefan Reisch	Hex: PS-c
• 2000 Clay Math Inst	P vs NP: $1 000 000

HUMANS

SOLVED OPENINGS

2001 Yang	17/49 7x7
2002 Yang	8x8
2003 Yang	9x9
2004 Noshita	7x7
2005 Noshita	8x8
2006 Mishima	8x8

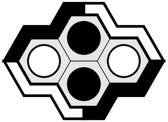
COMPUTERS

SOLVED OPENINGS

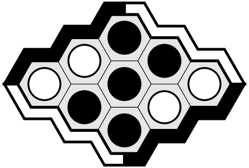
1995	Enderton		6x6
2000	van Rijswijck		6x6
2003	H Bjö Joh Kan Po vRij	5d	7x7
2007	Rasmussen et al.		7x7
2009	Arneson H Henderson	4d	8x8
2010	A H H	25d	some 9x9
2012	Pawlewicz H	110d x 24 thread	9x9
2013	Pawlewicz H	63d x 24 thread	centre 10x10

COMPUTERS

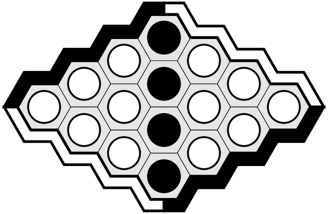
COMPUTERS



computers



COMPUTERS



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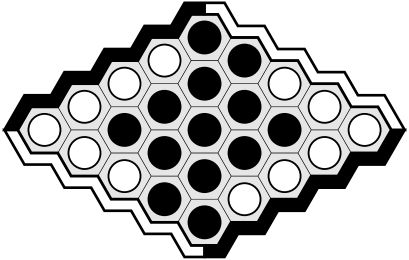

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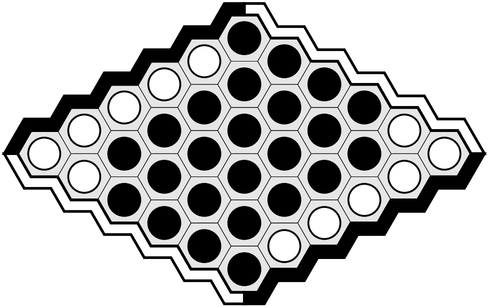

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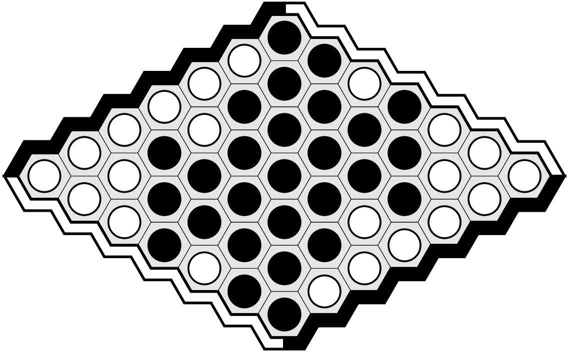

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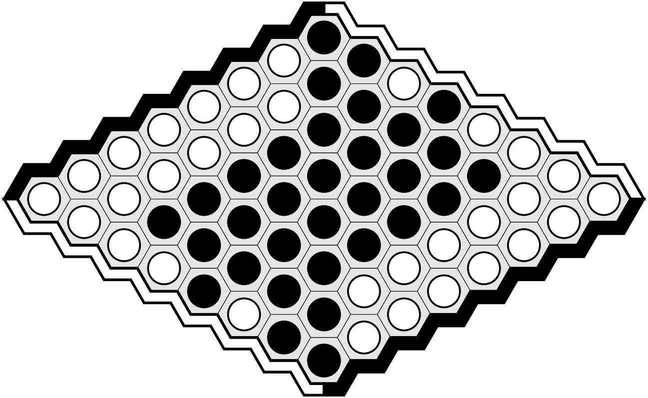

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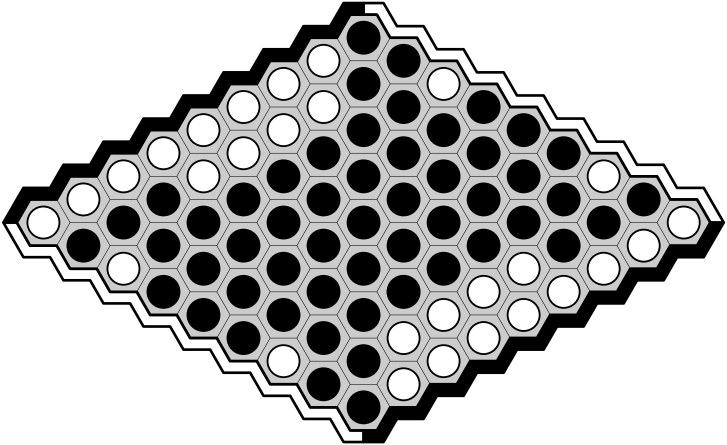
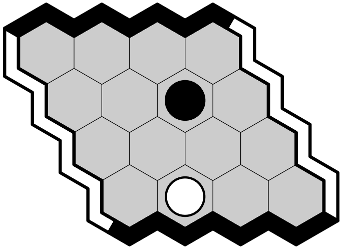


KNOWLEDGE

- virtual connections: combining rules, mustplay
- inferior cells: dead, captured, etc.

A VIRTUAL CONNECTION



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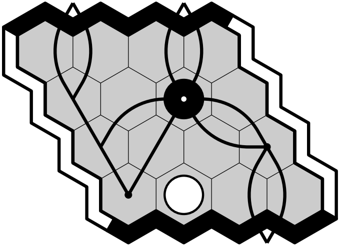
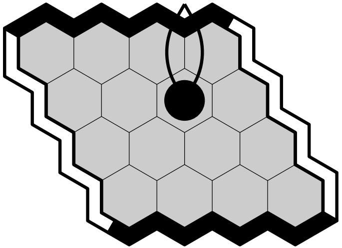


COMBINING RULE: AND (FULL)



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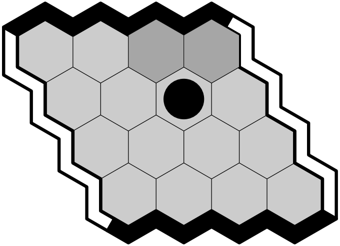

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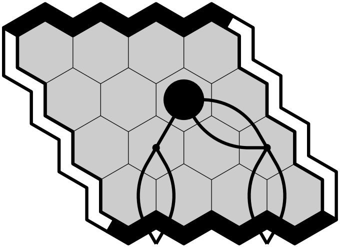

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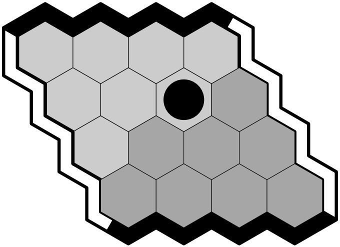

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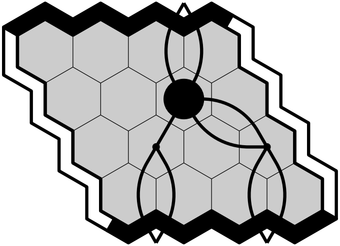

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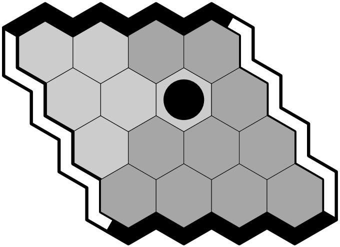
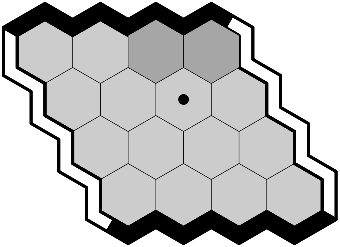


COMBINING RULE: AND (SEMI)



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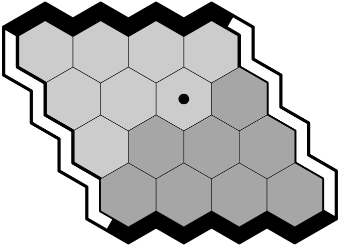

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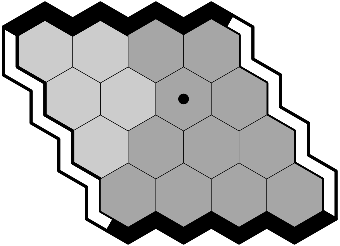
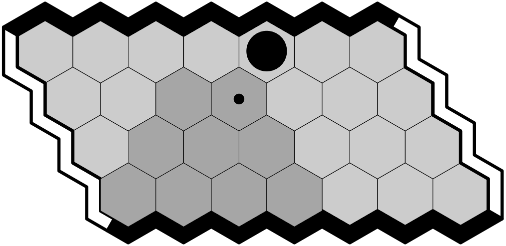


COMBINING RULE: OR



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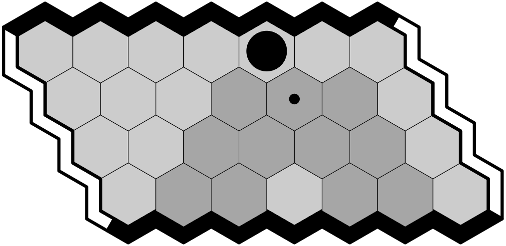

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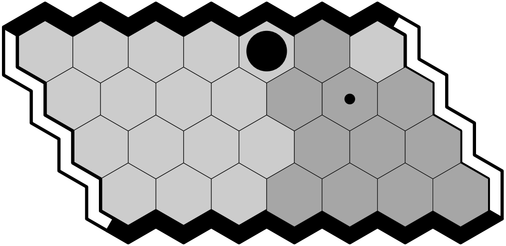

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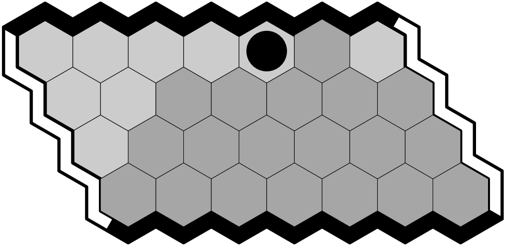
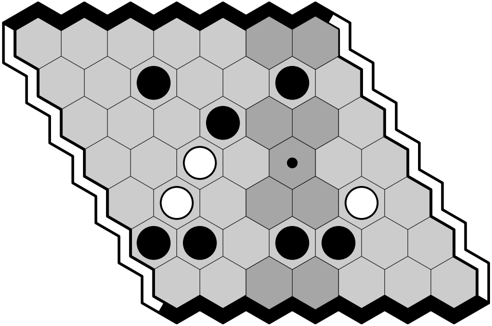


WHERE MUST WHITE PLAY?



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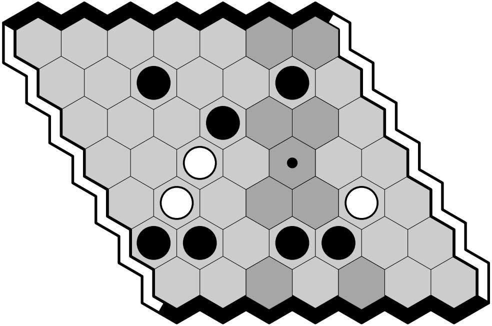

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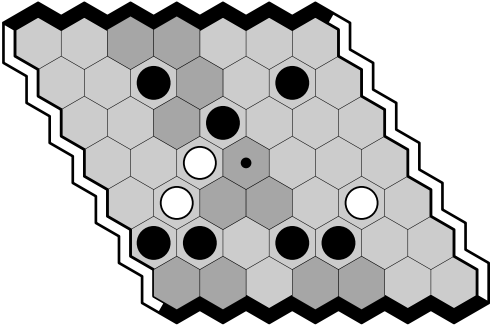

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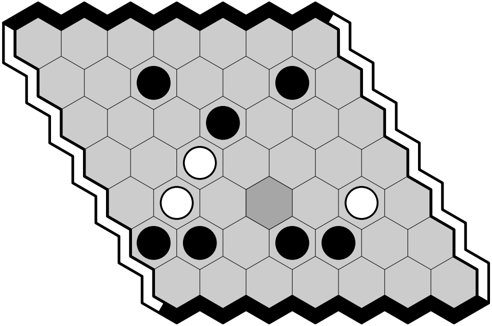
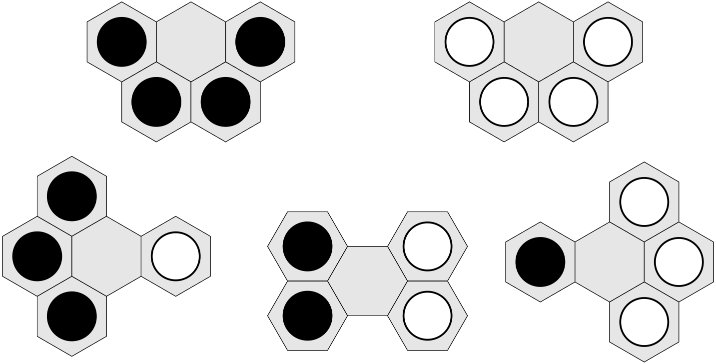
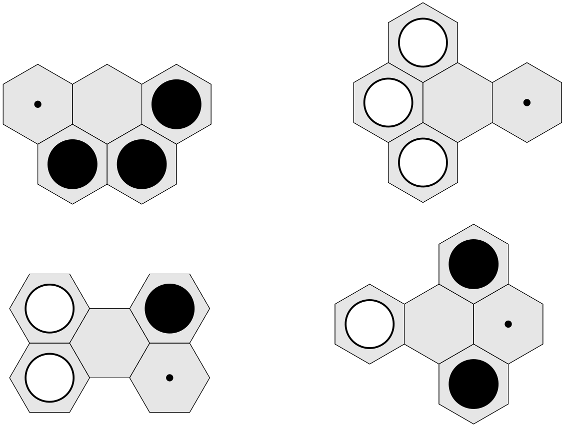


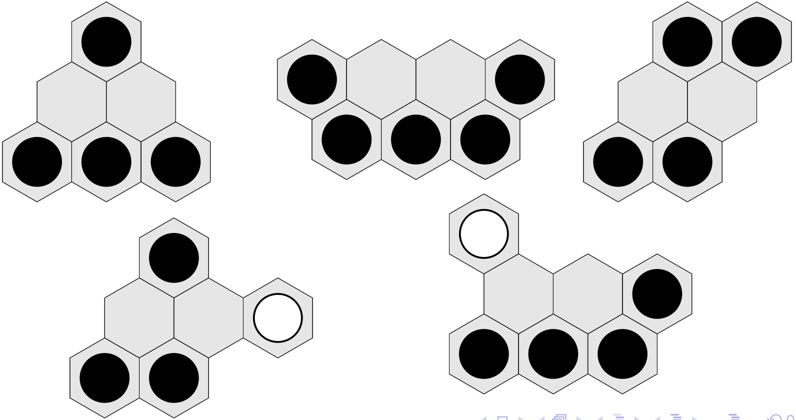
DEAD



BLACK-DOMINATED (DOT SUPERIOR)



BLACK-CAPTURED



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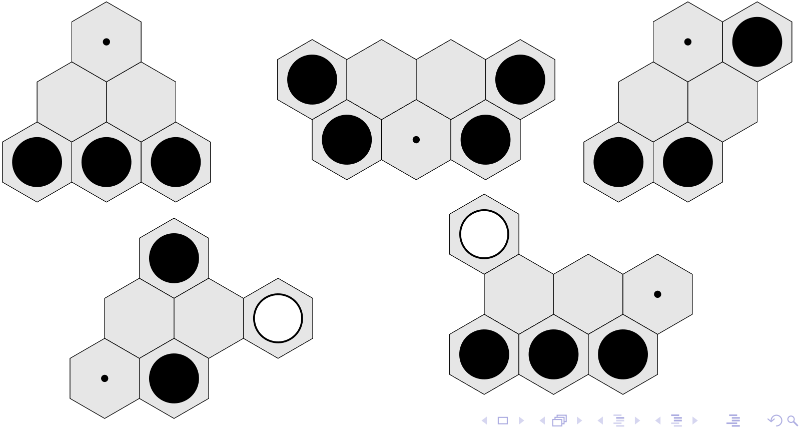
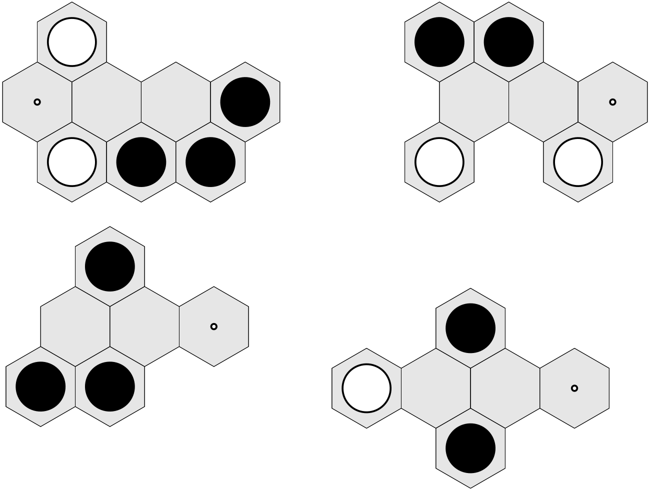
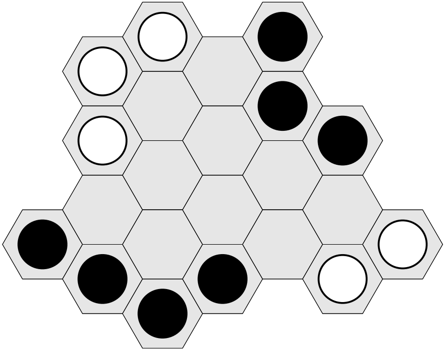


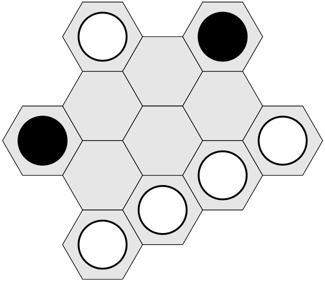
BLACK-CAPTURE-REVERSIBLE (TO WHITE DOT)



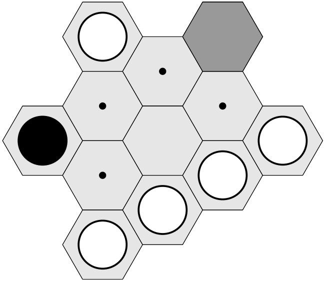
BLACK FILL DECOMPOSITION



STAR DECOMPOSITION

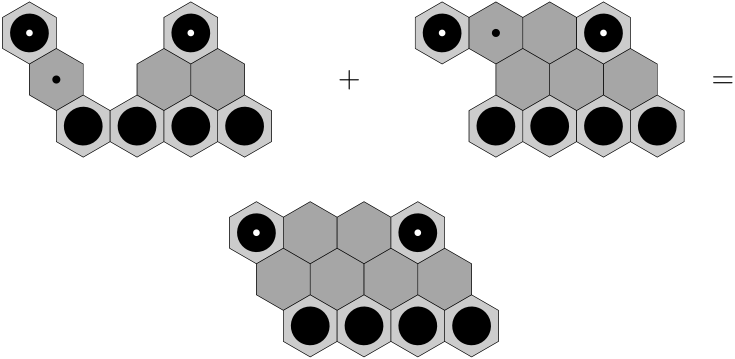


BLACK STAR DECOMP DOMINATION

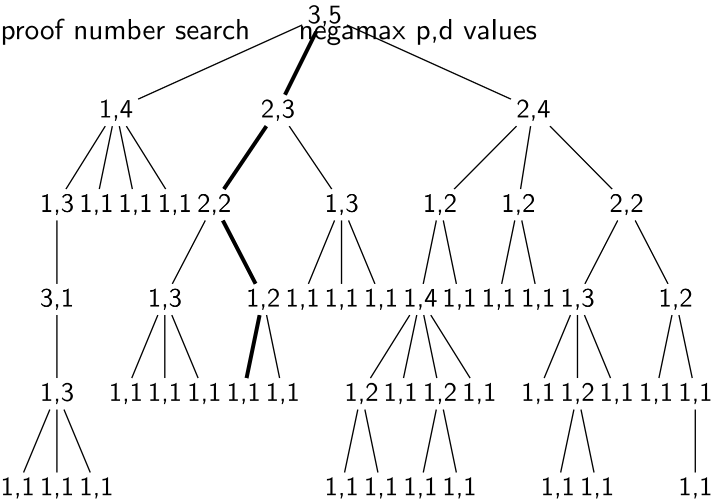


modify H-search

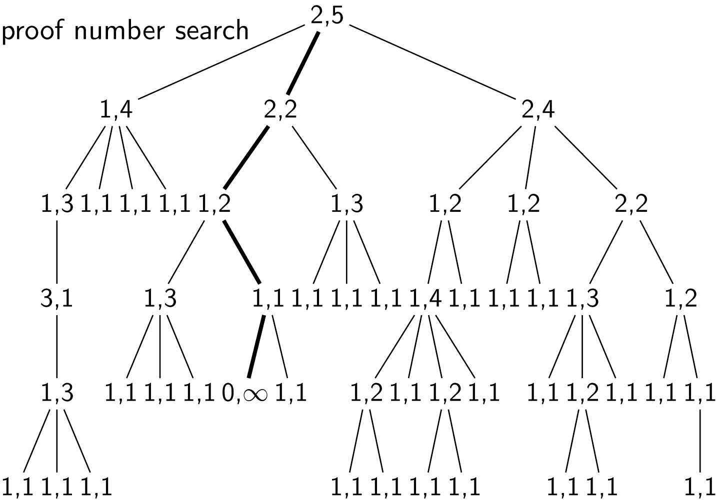
- and/or combining rules + capture



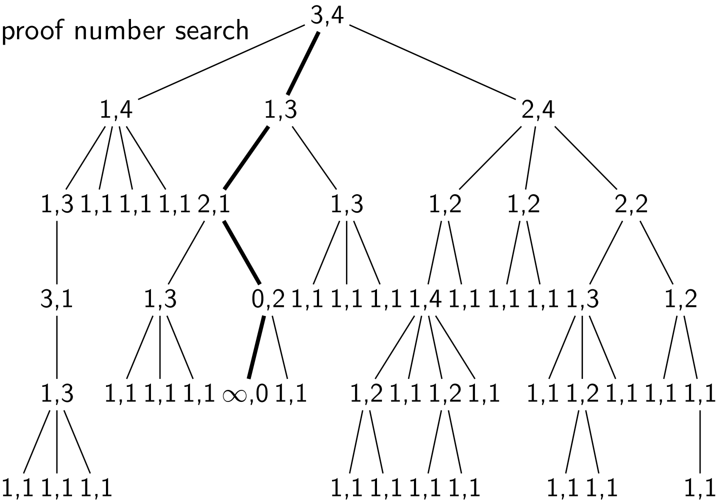
proof number search negamax p,d values



proof number search



proof number search



F-DFPNS

- PNS   Allis et al

F-DFPNS

- PNS Allis et al
- DFPNS Nagai

F-DFPNS

- PNS Allis et al
- DFPNS Nagai
- DFPNS in Hex ?

F-DFPNS

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- . . . requires non-incremental H-search :(

F-DFPNS

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- DFPNS Nagai
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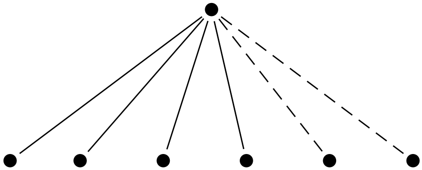
- . . . uniform branching factor :(

F-DFPNS

- PNS Allis et al
- DFPNS Nagai
- DFPNS in Hex ?
- ...requires non-incremental H-search :(
- ...uniform branching factor :(
- idea: move ordering + DFPNS = F-DFPNS

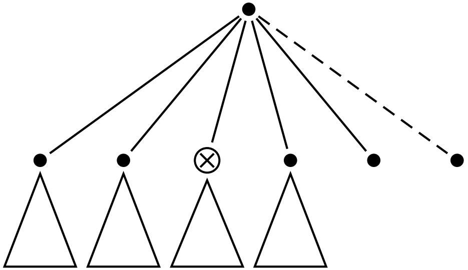
F-DFPNS (1)

- expand node
- consider first $b + \lceil f \times 6 \rceil = 4$ (of 6) live children



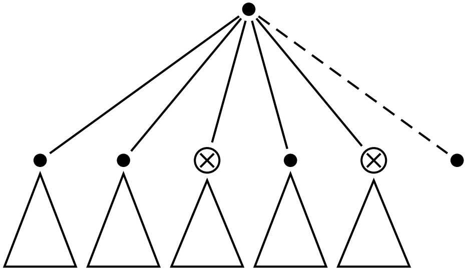
F-DFPNS (2)

- discover move 3 loses
- consider first $b + \lceil f \times 5 \rceil = 4$ (of 5) live children



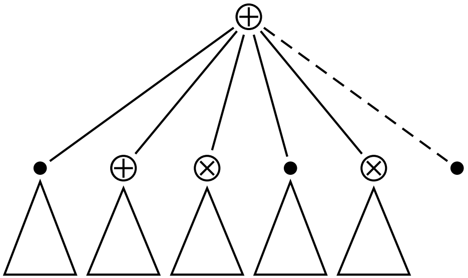
F-DFPNS (3)

- discover move 5 loses
- consider first $b + \lceil f \times 4 \rceil = 3$ (of 4) live children



F-DFPNS (4)

- discover move 2 wins, so . . .
- . . . root solved without exploring 6th move



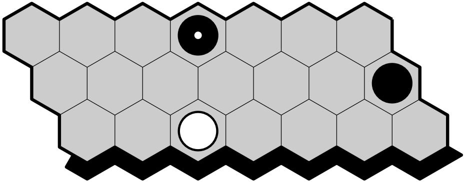
solving 10x10

- stronger VC computations
- scalable parallel DFPN S

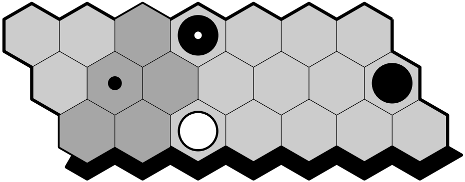
Pawlewicz: stronger VC computations

- faster and/or-rule VC computation
- limit form of new VCs, so never redundant
- find fewer VCs, but solve 2 to 10 times faster

Example: VCs to side



EXAMPLE: SEMIS



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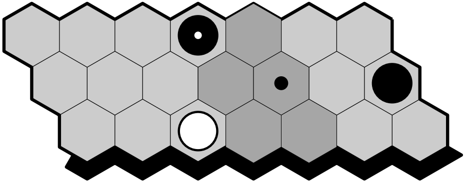

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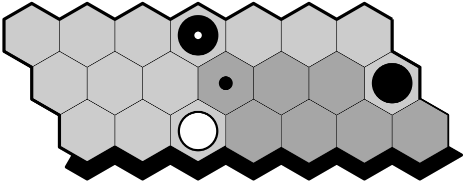

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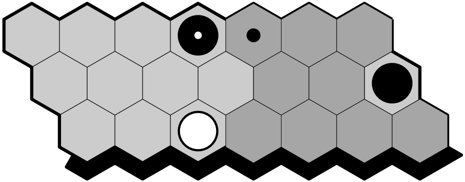
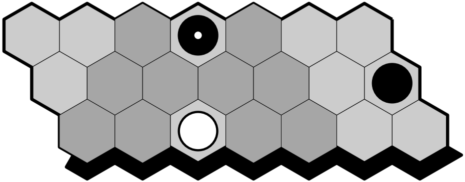
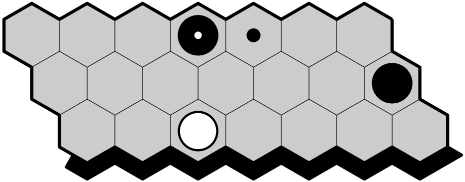


GREEDY UNION SEMIS TO GET FULL



Block cell to get another VC



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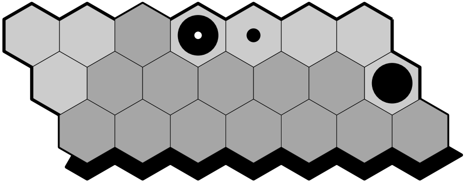
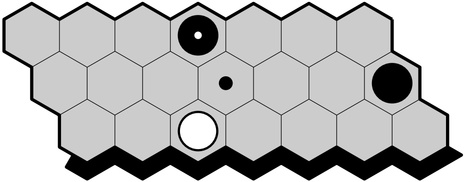
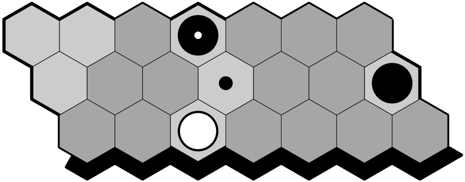


BLOCK CELL TO GET ANOTHER VC



BLOCK CELL TO GET ANOTHER VC



Pawlewicz: scalable parallel DFPNS

- parallel PNS: keep tree in memory? e.g. I-Chen Wu connect6
- Hex: leaf computations fast, so tree too big
- how to assign jobs to processors?
  - jobs too long: computation redundant
  - jobs too short: too much client/server traffic
  - solution: MaxWorkPerJob

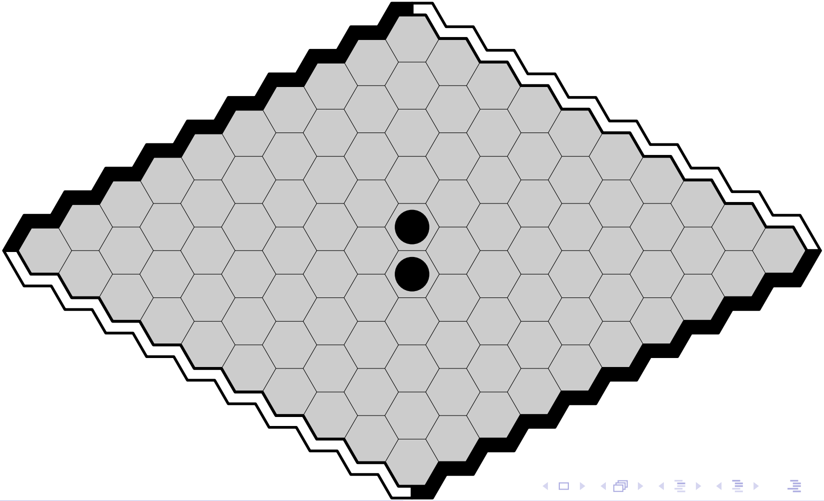
SP DFPN S features

- 1+ε variant of DFPN
- advanced TT resolution: upon collision, search next k (say 4) cells for empty location; if none found, overwrite location with smallest work job
- once node computation assigned to leaf, use virtual win/loss so new threads go elsewhere
- ...so compute virtual (dis)proof numbers
- shared TT: many-read / 1-write locks
- tune MaxWorkPerJob
- for Hex: use Focussed DFPN S

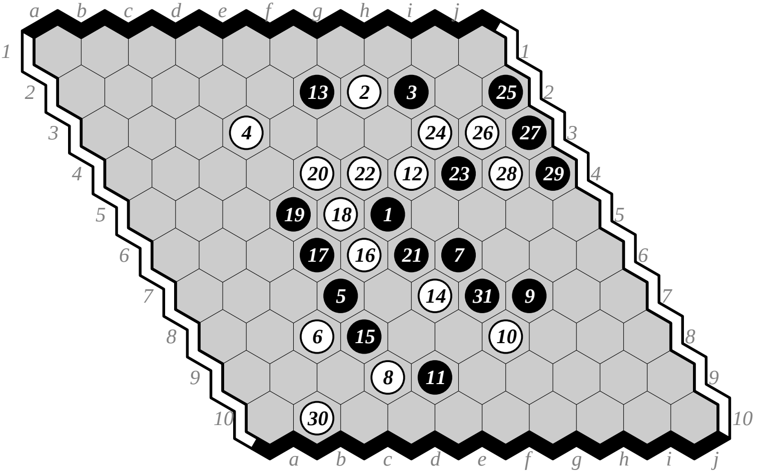
SP DFPN S performance

- speedup test: 8 hardest 8x8 openings, 8 11x11 positions
- speedup performance: 11.8 on 16 threads (.74)
- solved all 9x9 openings
- solved centre 10x10 opening

SP DFPN S PERFORMANCE



SP DFPN S performance



SP DFPN S performance

opening	#threads	time	winner
a2	8/24	68d09:40:18	black
a3	8	80d08:37:34	white
a4	8	33d14:06:03	black
a5	8	65d04:14:52	black
a6	24	110d14:35:06	black
a7	24	4d08:56:03	white
a8	24	6d14:21:30	black

SP DFPN S performance

opening	#threads	time	winner
b2	8	53d15:18:21	black
b4	8	29d23:53:14	black
b6	8	1d21:52:28	black
b7	8	4d17:19:13	black
c2	24	1d08:42:57	black
i1	24	6d00:51:25	black
10x10:f5	24	63d20:44:30	black

HOW LONG UNTIL 11X11 ?

	states (approx)	center cell: solver fn calls
2x2	9.0 e 0	0
3x3	5.5 e 1	0
4x4	7.6 e 5	0
5x5	4.0 e 9	0
6x6	4.0 e 14	2
7x7	1.5 e 20	68
8x8	1.0 e 27	19 554
9x9	2.7 e 34	912 352
10x10	1.2 e 43	5 821 097 789
11x11	2.2 e 52	??? ??? ??? ??? ???

THANK YOU

- invitation Prof Mark Kayll UMontana
- solving 10×10 Hex joint with Jakub Pawlewicz
- builds on work with B Arneson, P Henderson
- machine Martin Müller
- photo courtesy MIT Museum, MIT, Cambridge MA
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