Monophonic Intervals and the Game of Hex

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Thankyou ...

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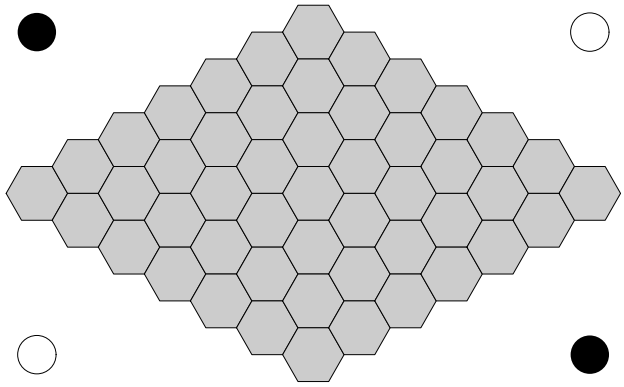
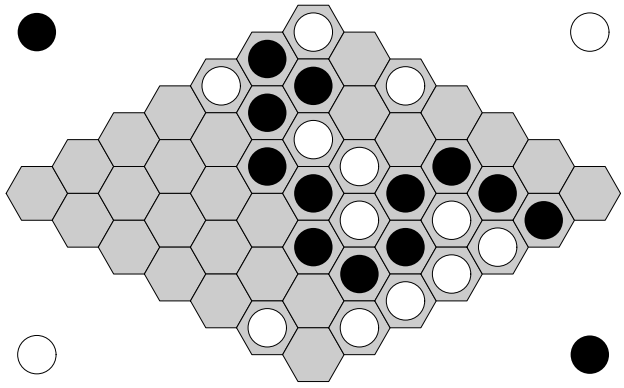
UofA Games Research Group

the game of Hex





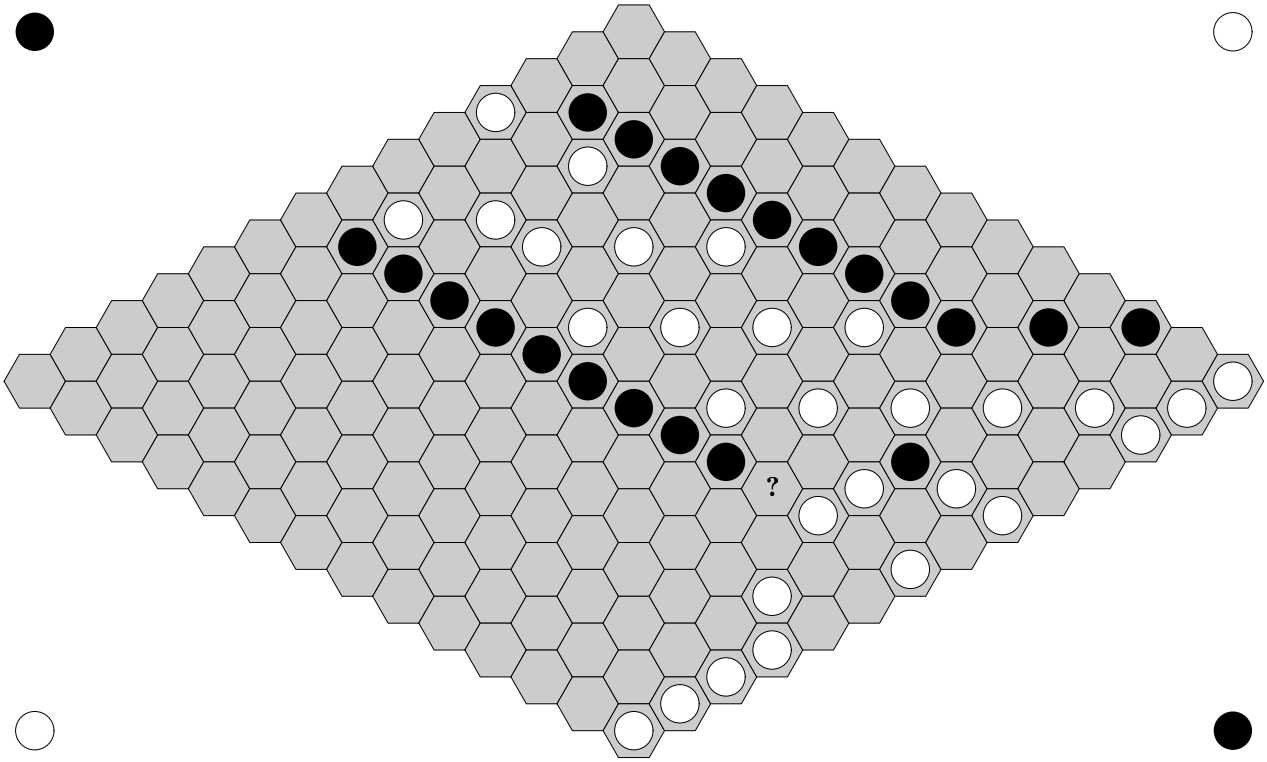
'42	**Piet Hein**	**Copenhagen**
'48	**John Nash (& David Gale)**	**Princeton**
'53	**Claude Shannon & E.F. Moore**	**Murray Hill**
'57	**Martin Gardner**	**? Hendersonville NC**
'69	**Anatole Beck**	**Madison WI**
'75	**Craige Schensted & Ch' Titus**	**Peaks Is. ME**
'76	**Shimon Even & Robert Tarjan**	
'77,'81	**Claude Berge**	**Paris**
'79	**David Gale**	**Berkeley CA**
'81	**Stefan Reisch**	**? Germany**
'91	**Sid Sackson & Klutz Press**	**Palo Alto CA**
'00	**Jack van Rijswijck**	**Edmonton AB**
'00	**Cameron Browne**	**Brisbane**
'00	**Vadim Anshelevich**	**Richardson TX**
'01	**Jing Yang**	**Winnipeg MB**
'02	**H**	**Edmonton AB**
'03	**H B J K P vR**	**Edmonton AB**
'04	**H vR**	**Edmonton AB**

Claude Berge

- *L'Art Subtil du Hex* **'77**
- *Some remarks about a Hex problem ...* **'81**

It would be nice to solve some Hex problem by using nontrivial theorems about combinatorial properties of sets.

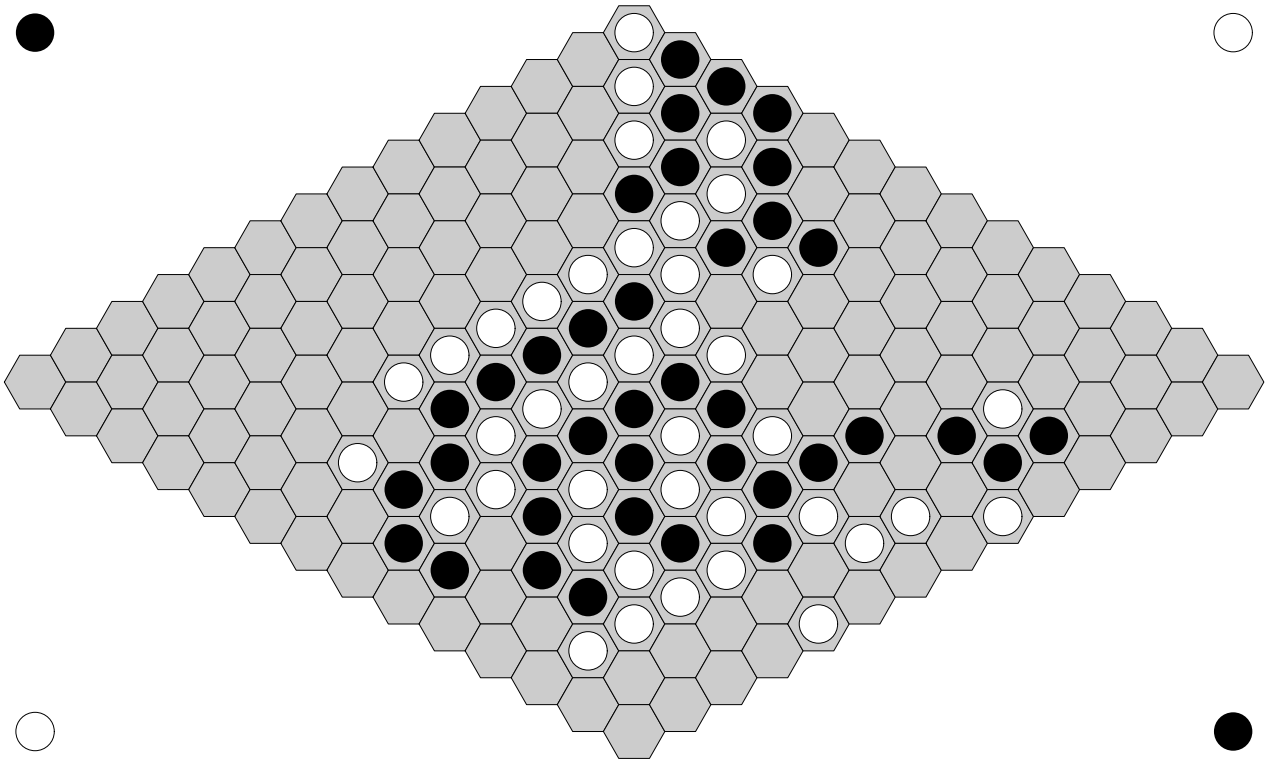
Berge puzzle: Black to play and win



• Berge and the Art of Hex H '02

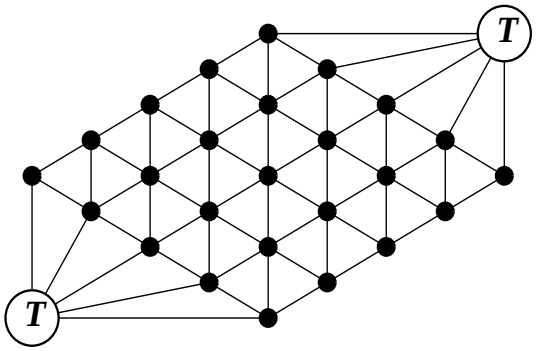
mystery of the missing stone
(and other stories)

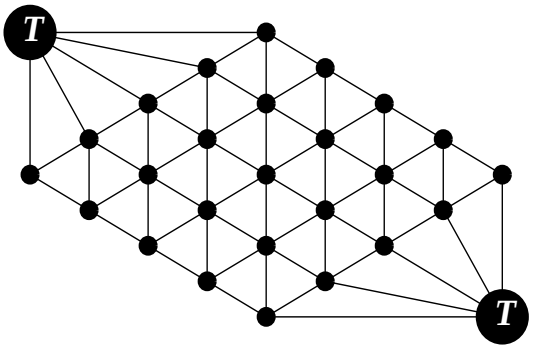
White to play and win

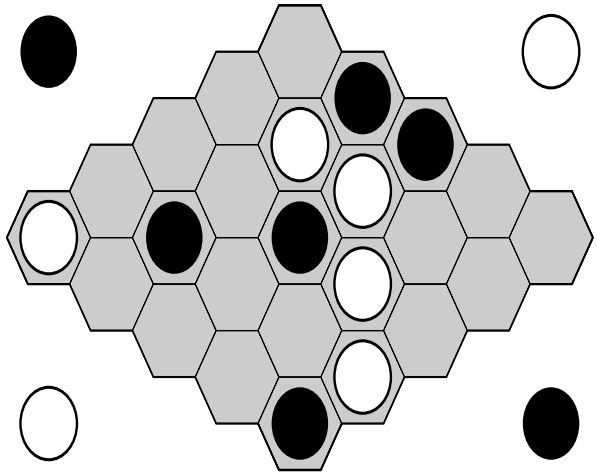


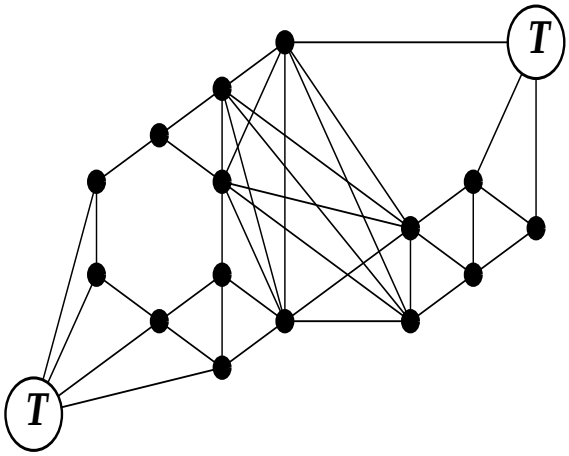
monophonic intervals

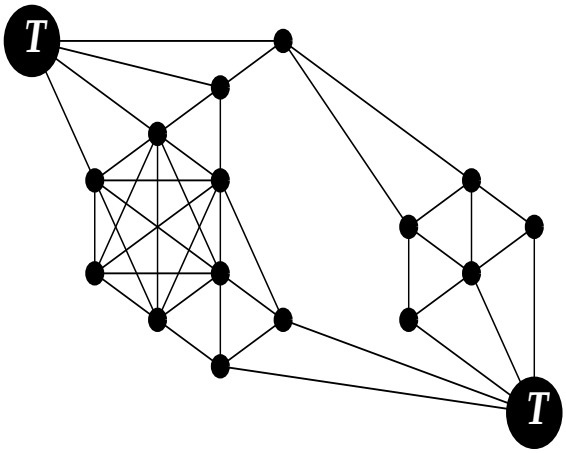
- **node v is *dead* if,**
  for every completion of $G - v$,
  colour of v does not change winner
- ***live* iff not dead**
- **Theorem: live iff on terminal-terminal monophonic interval of reduced graph**











- **computing m. i. NP-hard (Fellows)**
- **dead nodes often simplicial**

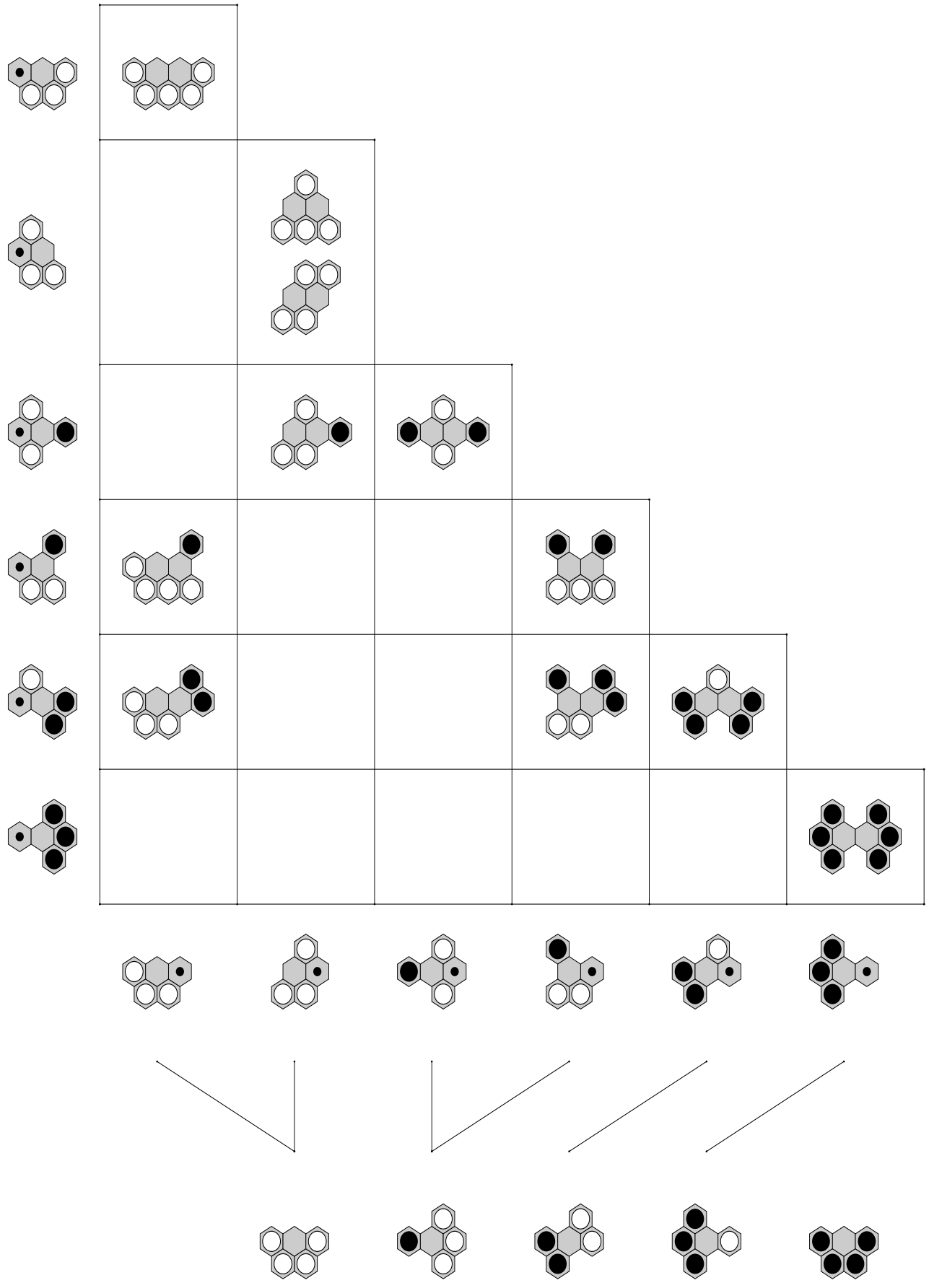
death has consequences

- **P-captured set of nodes:**

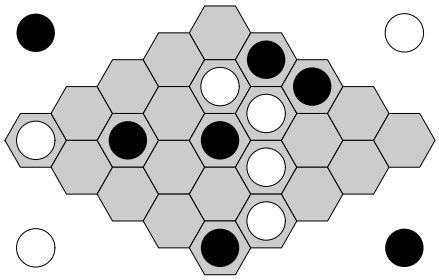
  adding P-stones doesn't change game

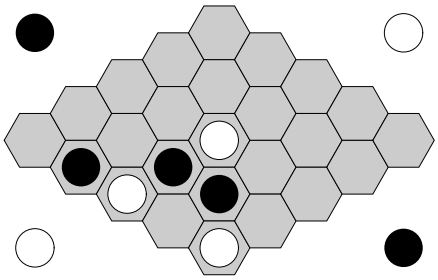
- **P-dominated set of nodes:**

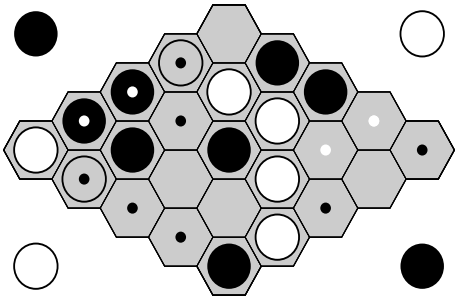
  some P-move in set P-captures the rest;
  such a move is P-dominating;
  P can ignore all other moves into the set

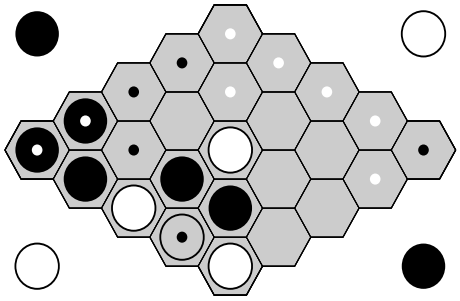


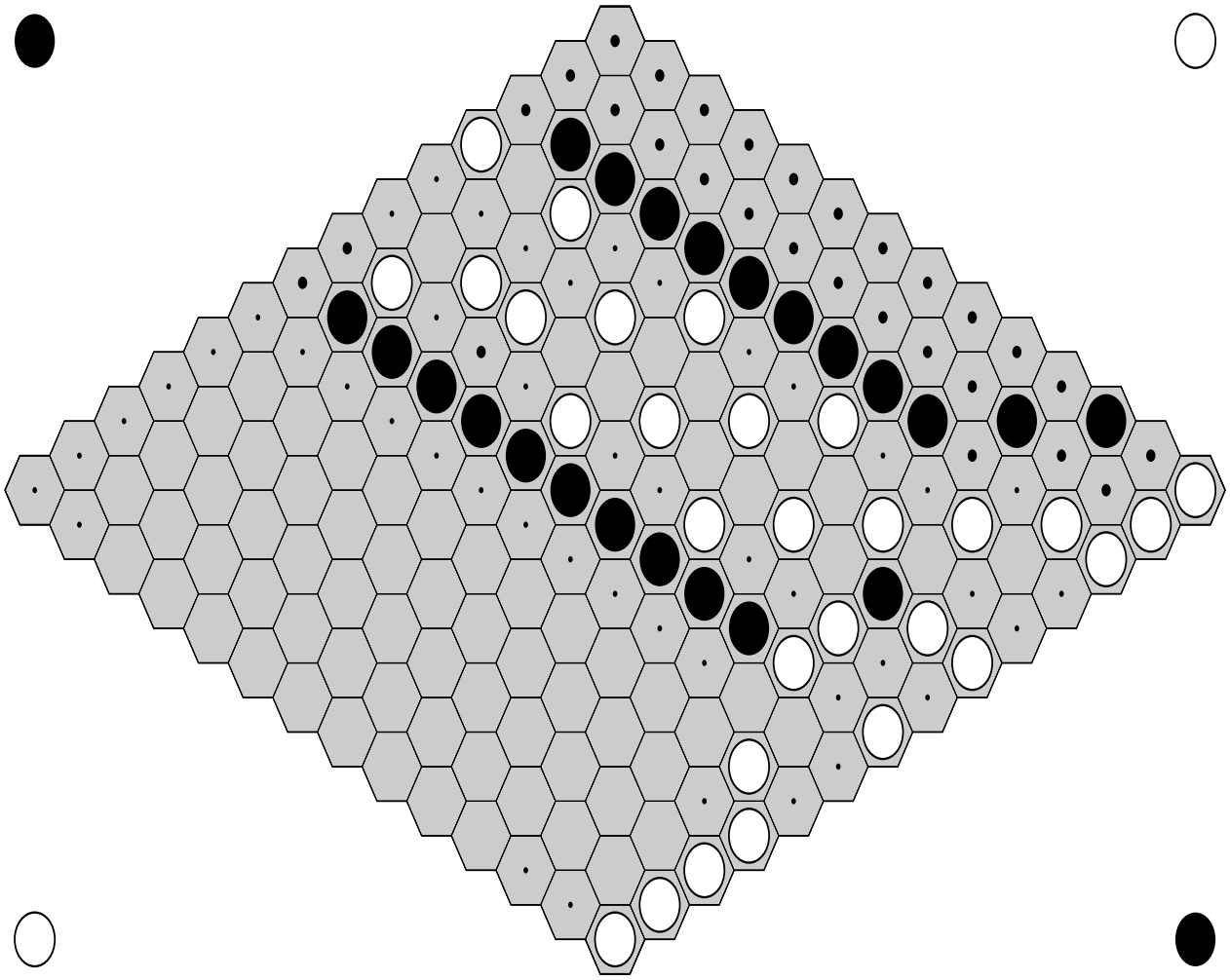
dead cell analysis

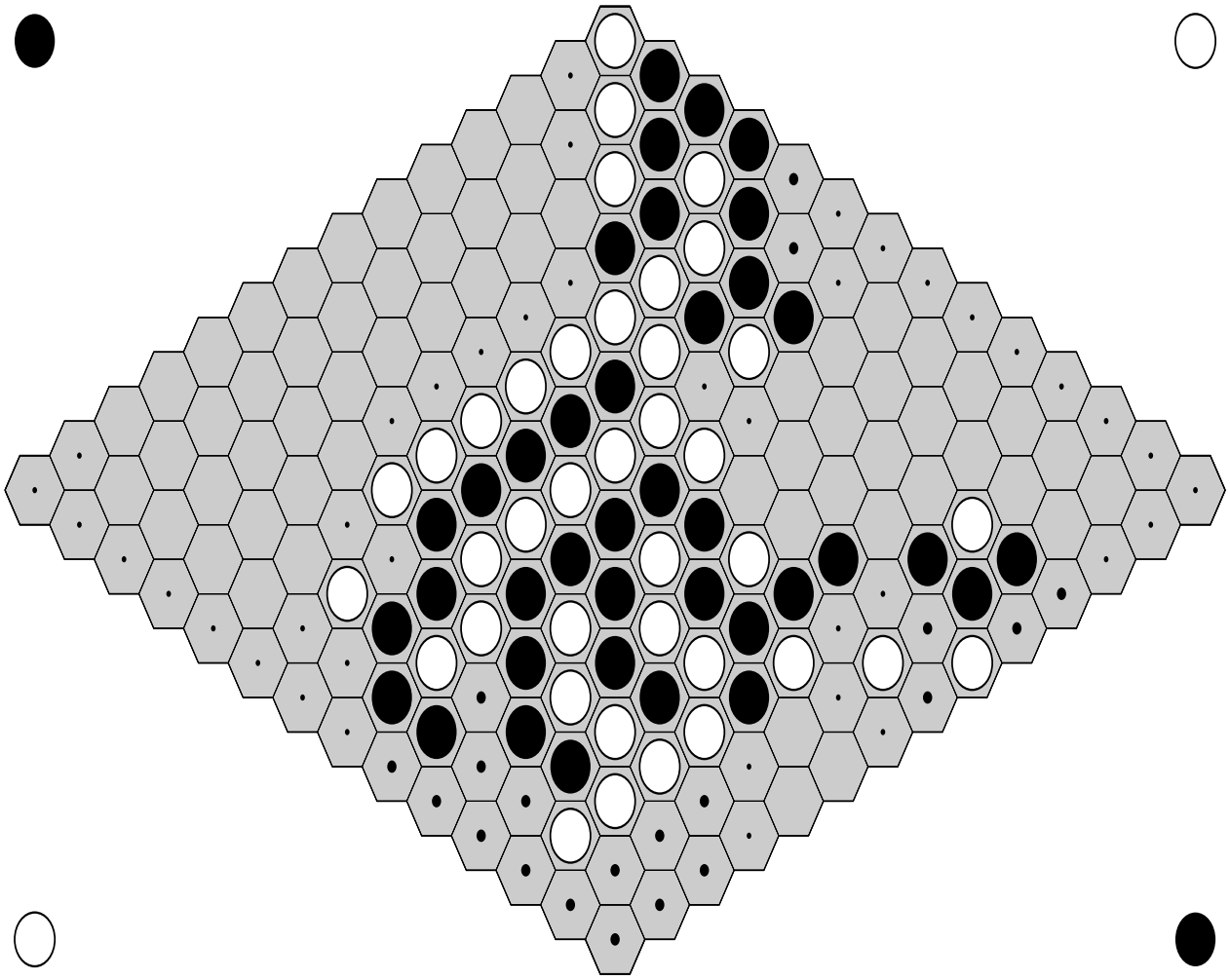












A set S of unoccupied nodes is

P-captured:

if S is empty, or
for each opponent-move to m in S

- **$S - m$ is P-dominated, and**
- **filling $S - m$ with P-stones makes m dead**

P-dominated:

if S is empty, or
there is some P-move to m in S so that

- **$S - m$ is P-captured**