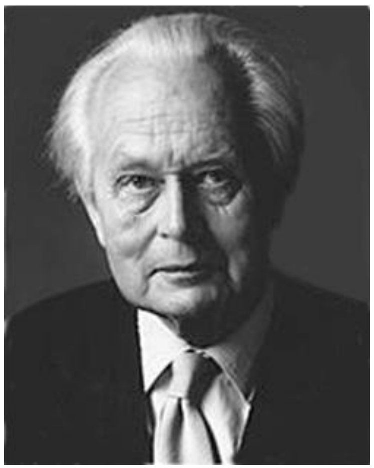
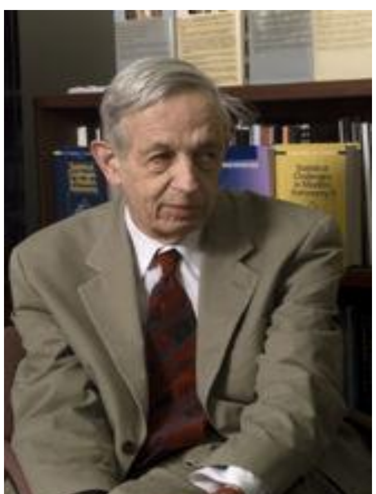
Piet Hein and John Nash: BEAUTIFUL MINDS

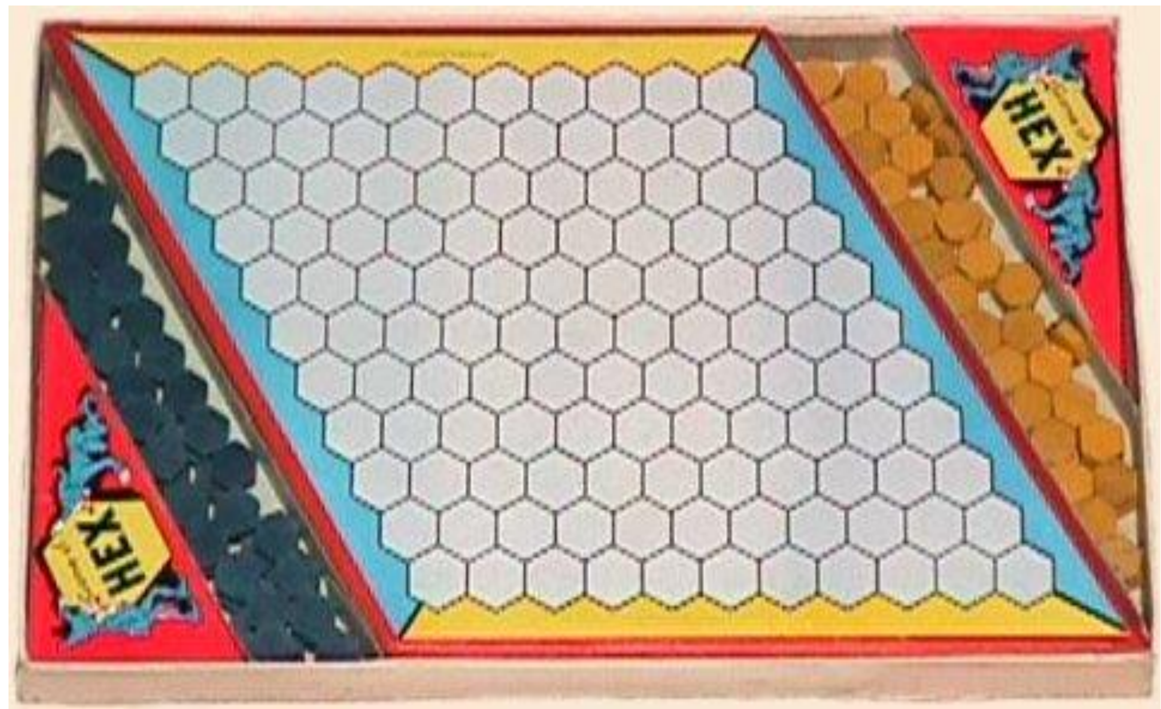
Talk by Bjarne Toft, University of Southern Denmark

Piet Hein
1905-1996



John Nash
1928-2015





High School Graduation 1924
The lost spring

Copenhagen Conference 1932

Heisenberg, Werner Karl; Hein, Piet; Bohr, N.; Brillouin, Leon Nicolas; Rosenfeld, Leon; Delbrück, Max; Heitler, Walter; Meitner, Lise; Ehrenfest, Paul; Bloch, Felix; Waller, Ivar; Solomon, Jacques; Fues, Erwin; Strømgren, Bengt;
Kronig, Ralph de Laer; Gjelsvik, A; Steensholt, Gunnar; Kramers, Hendrik Anton; Weizsäcker, Carl Friedrich von;
Ambrosen, J.P.; Beck, Guido; Nielsen, Harald Herborg; Buch-Andersen; Kalckar, Fritz; Nielsen, Jens Rud; Fowler,
Ralph Howard; Hylleràs, Egil Andersen; Lam, Ingeborg; Rindal, Eva; Dirac, Paul Adrian Maurice; N.N.; Darwin, Charles Galton; Manneback, Charles; Lund, Gelius

Piet Hein to Martin Gardner (1957)

1.

POLITIKEN

TELEGRAM-ADR.: POLITIKEN, KØBENHAVN
POLITIKENS HUS

TLF.: CENTRAL 8511, RIGS 56, TELEX 2341
KØBENHAVN V

REDAKTIONEN

June 24, 57

Discription of

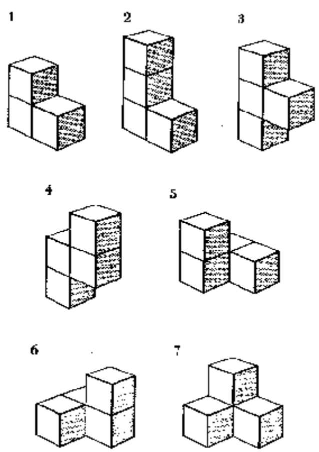
THE SOMA CUBE

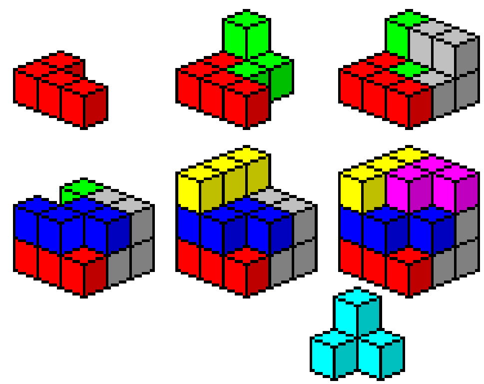
During a lecture in quantum physics by one very dry and systematisch middle European physitist – dealing with a space (6-dimensional at that) cut up in cubicles I fell asleep (please observe I don't pretend this to be a rare experience) — and had a revelation:

. . . . .

If you take all „elements of orthogonal connection" they can be put together to the orthogonal unit again. I woke up – and Heisenberg was still talking! – and tried the revelation on paper, and it proved true. – The variations of the unit – which combine to the unit again ... That is the smallest philosophical system of the World – and smallnes is no small quality in a philosophical system

Soma – a contradictory surprise





SOMA in 1933

SOMA
ENG. P.
420349
KR. 1,50
D.R.G.M.
1321938
DANSK PATENT ANMELDT
SOMA er 1000 Spil i 1.
SOMA - Diamanten.

Piet Hein discovered Hex in 1942

Parentesen, Copenhagen University, December 1942

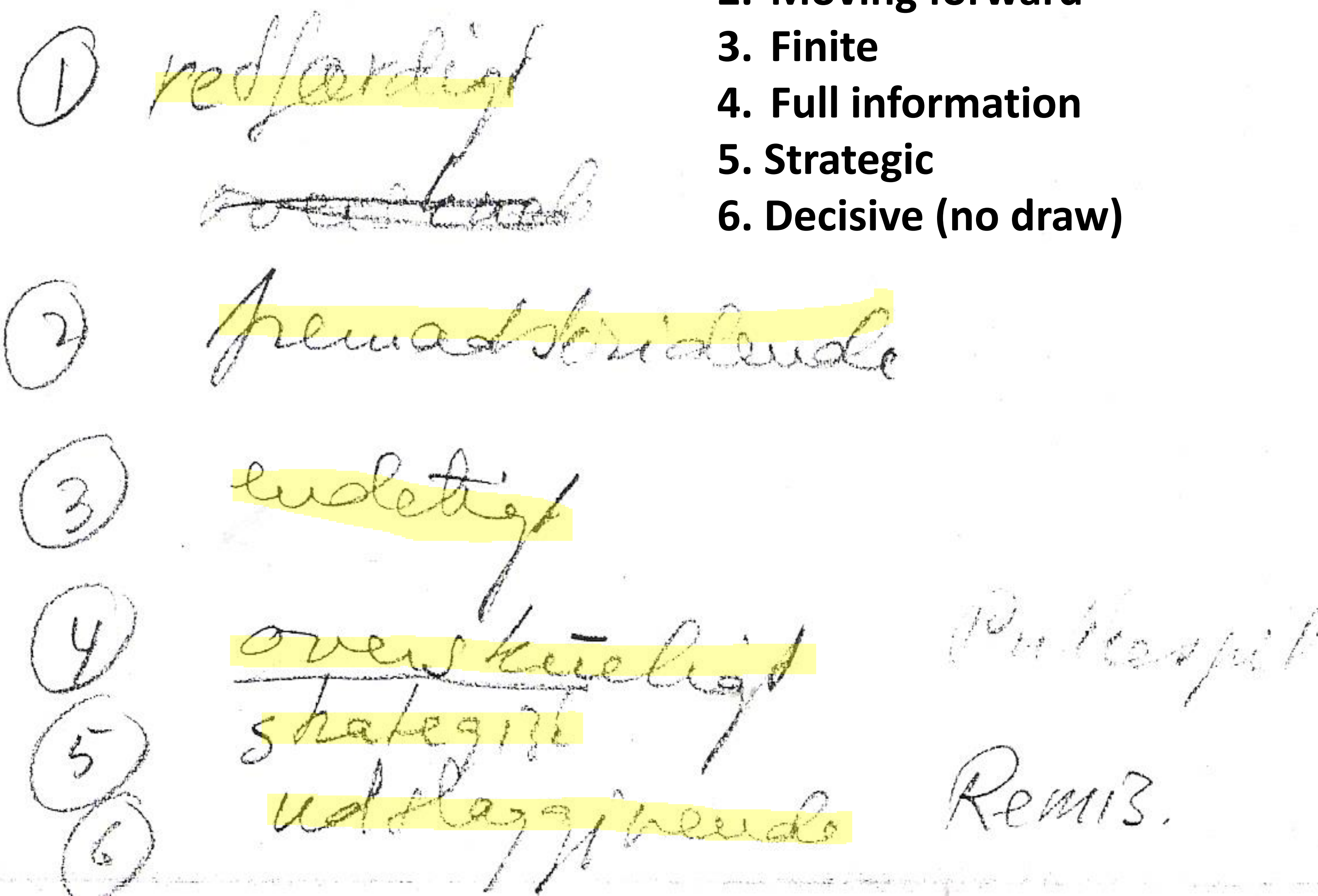
The Mathematics of games and Games as mathematics

Tanken er at se paa Matematikken som Spil
Spillet er et Exempel paa at se paa Spil som Matematik.

Det jeg har at komme med i Aften er kun en Skitse til en Tanke som Indledning til et Spil. Jeg ved ikke, hvor meget aandelig Næring der er paa det for Dem, saa det vil berolige mig, hvis De vil fortsætte med at drikke og spise.

En litterær Anmelder af den Slags som - med Rette - ser deres egen Ophøjelse i at rakke ned paa den menneskelige Evne som kaldes Intelligens, srev for et par Aar siden i en Artikel om noget helt andet, at Matematik kan ikke give os andet end, hvad vi i Forvejen havde i Præmisserne. Det er jo rigtigt. Og det kaster et Skær over Matematikken af at være en ganske taabelig Virksomhed. Og i Artiklen fortsatte han da ogsaa som om Matematikken med denne Bemærkning en

1. Just
2. Moving forward
3. Finite
4. Full information
5. Strategic
6. Decisive (no draw)



Ikke [illegible] – men Papir og Blyant

Kronologien [illegible] kan være [illegible] et [illegible].

Den første kan vinde

The first can win

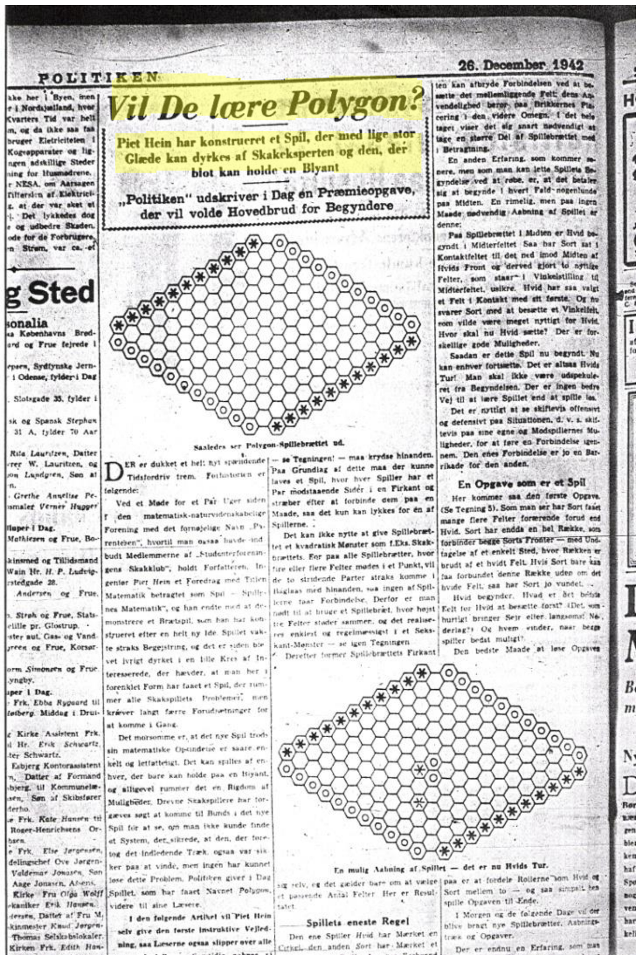
bevis i Meddelelse [illegible] Skak

And this can be proved

[illegible] trods den anden kunde vinde

[illegible]

Politiken Dec. 26, 1942



POLITIKEN 26. December 1942

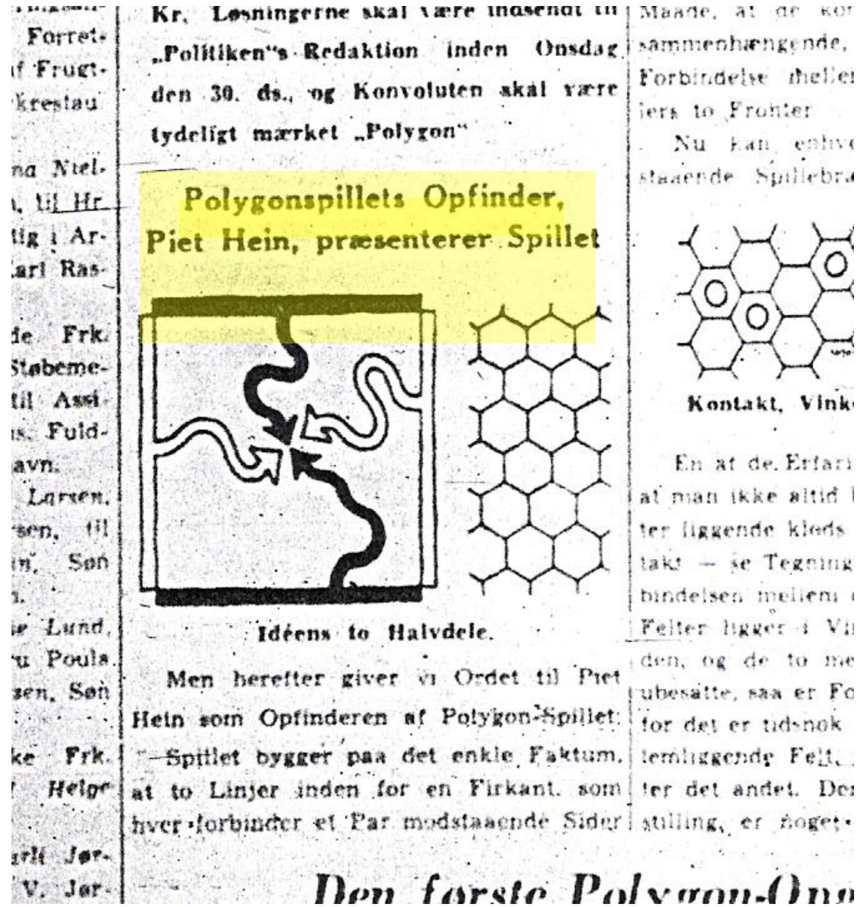
Vil De lære Polygon?

Piet Hein har konstrueret et Spil, der med lige stor Glæde kan dyrkes af Skakeksperten og den, der blot kan holde en Blyant

„Politiken" udskriver i Dag en Præmieopgave, der vil volde Hovedbrud for Begyndere

Saaledes ser Polygon-Spillebrættet ud.

En mulig Aabning af Spillet — det er nu Hvids Tur.



Kr. Løsningerne skal være indsendt til „Politiken"s Redaktion inden Onsdag den 30. ds., og Konvoluten skal være tydeligt mærket „Polygon"

Polygonspillets Opfinder, Piet Hein, præsenterer Spillet

Idéens to Halvdele.

Kontakt, Vink

Men herefter giver vi Ordet til Piet Hein som Opfinderen af Polygon-Spillet:

— Spillet bygger paa det enkle Faktum, at to Linjer inden for en Firkant, som hver forbinder et Par modstaaende Sider

Den første Polygon-Opg

Piet Hein Problems 1-46 from Politiken Dec.1942-June 1943

White to play.

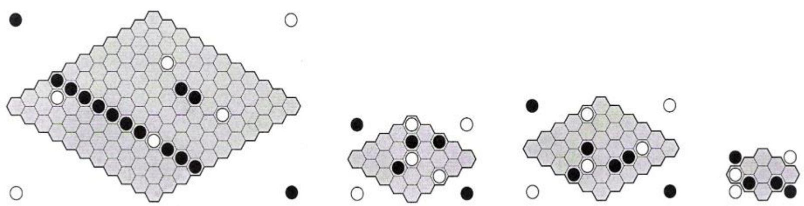


Fig. 1. Hein Puzzles 1–4

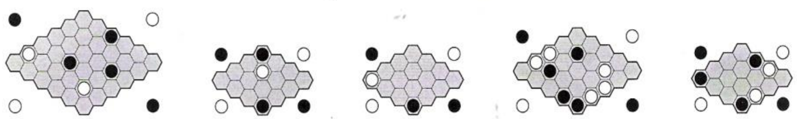


Fig. 2. Hein Puzzles 5–9

Fig. 3. Hein Puzzles 10–14

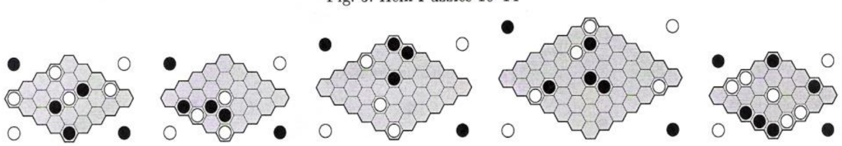


Fig. 4. Hein Puzzles 15–19

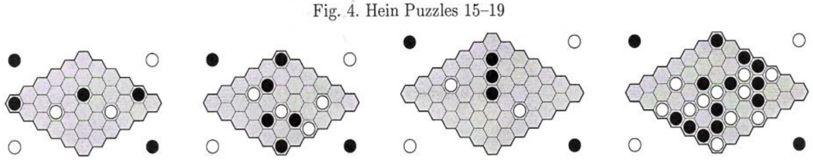


Fig. 5. Hein Puzzles 20–23

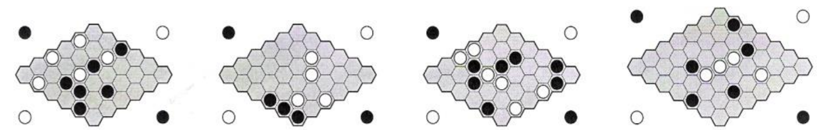


Fig. 6. Hein Puzzles 24–27

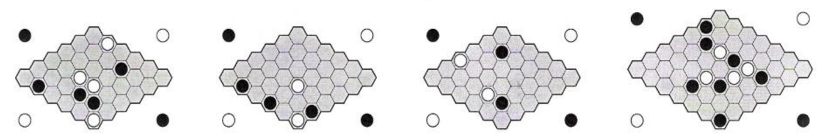


Fig. 7. Hein Puzzles 28–31

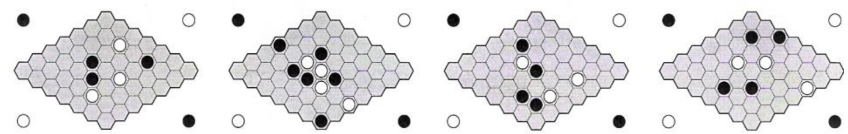


Fig. 8. Hein Puzzles 32–35

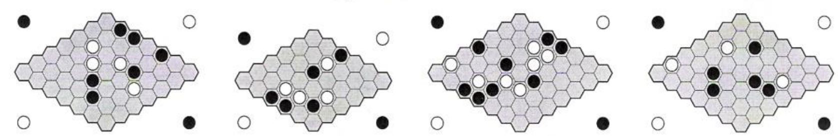


Fig. 9. Hein Puzzles 36–39

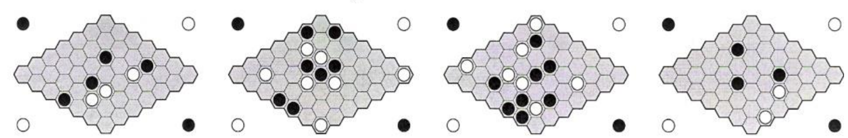


Fig. 10. Hein Puzzles 40–43

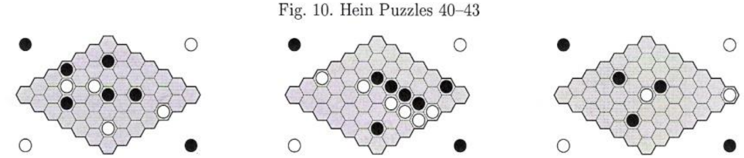
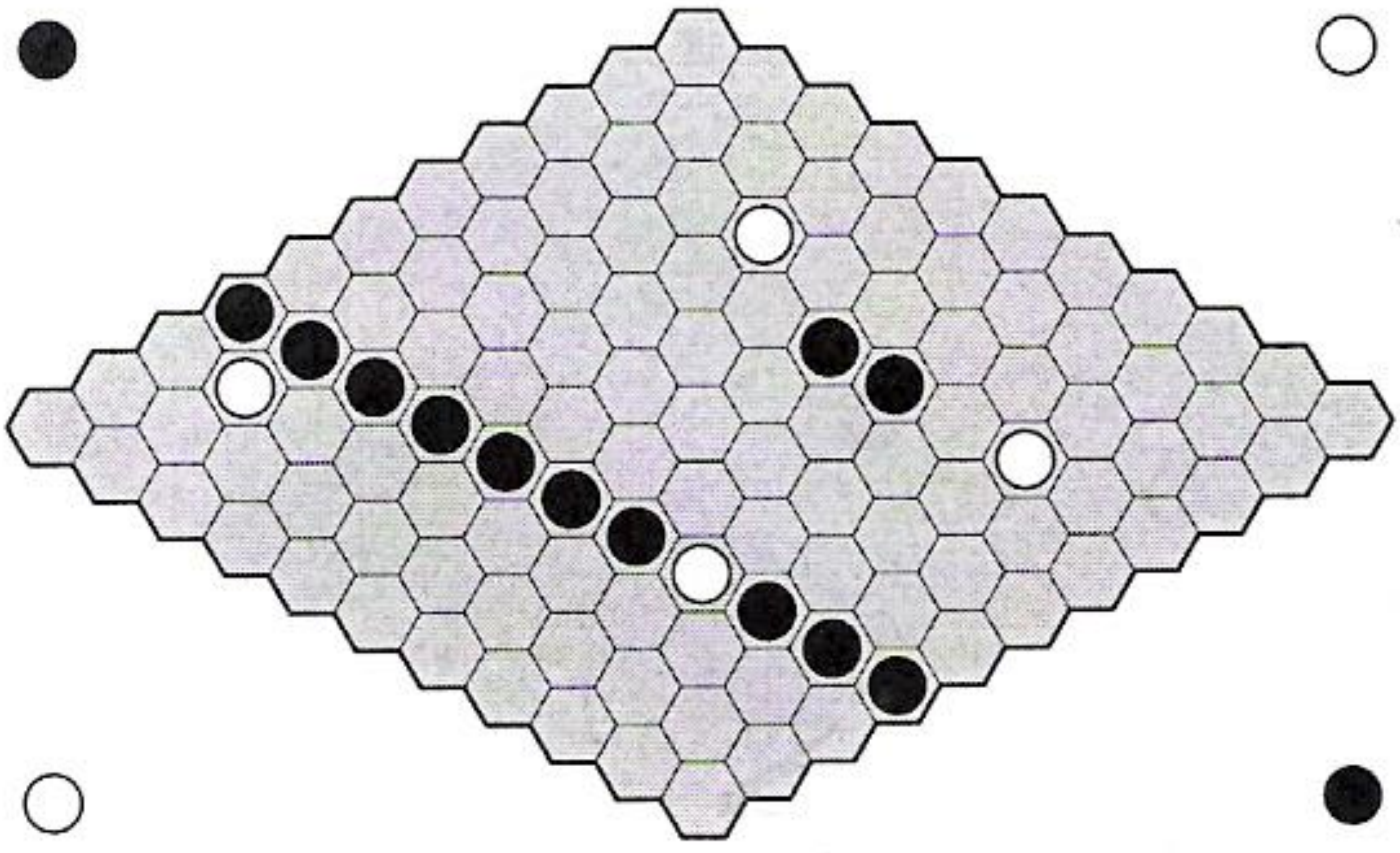
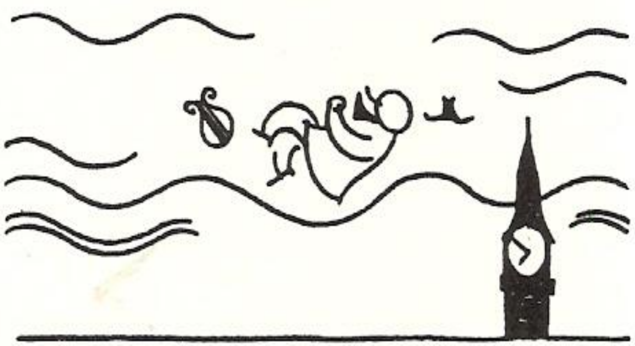


Fig. 11. Hein Puzzles 44–46

Piet Hein Problem 1
White plays and wins !



Life as a game of Hex



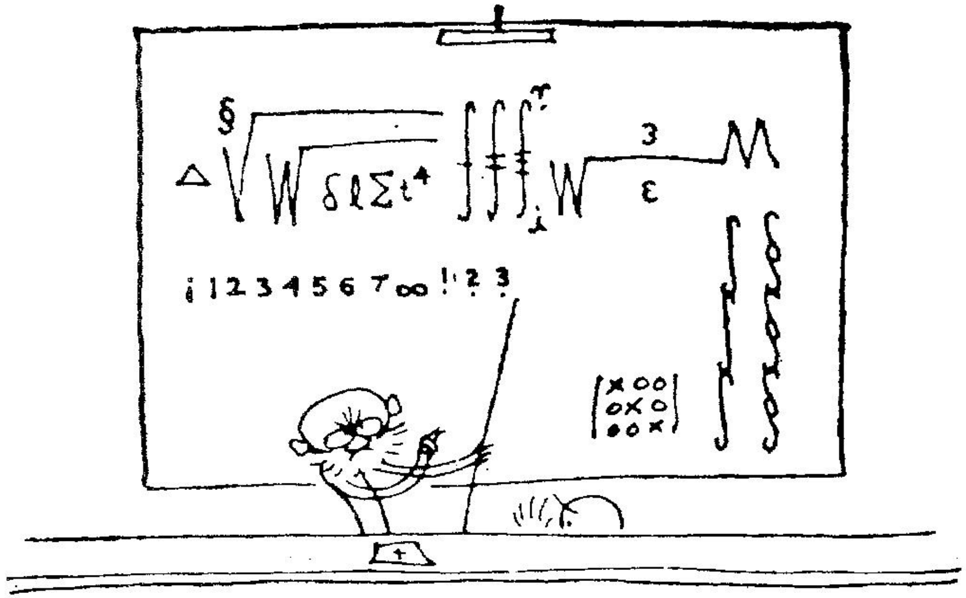
Life is almost
like a game
Easy – hard
Decide your aim
With the simplest
Rules you start
Most easy then
To make it hard.
(transl. BT)

LIVET BETRAGTET SOM SPIL

Gruk ved et spil Polygon.

Livet er ganske
som sådan et spil:
let eller svært
som man gør det til,
og: når de enkleste
regler er lært,
så er det lettest
at gøre det svært.

The road to wisdom? – Well, it's plain
and simple to express:
Err and err and err again
but less and less and less.

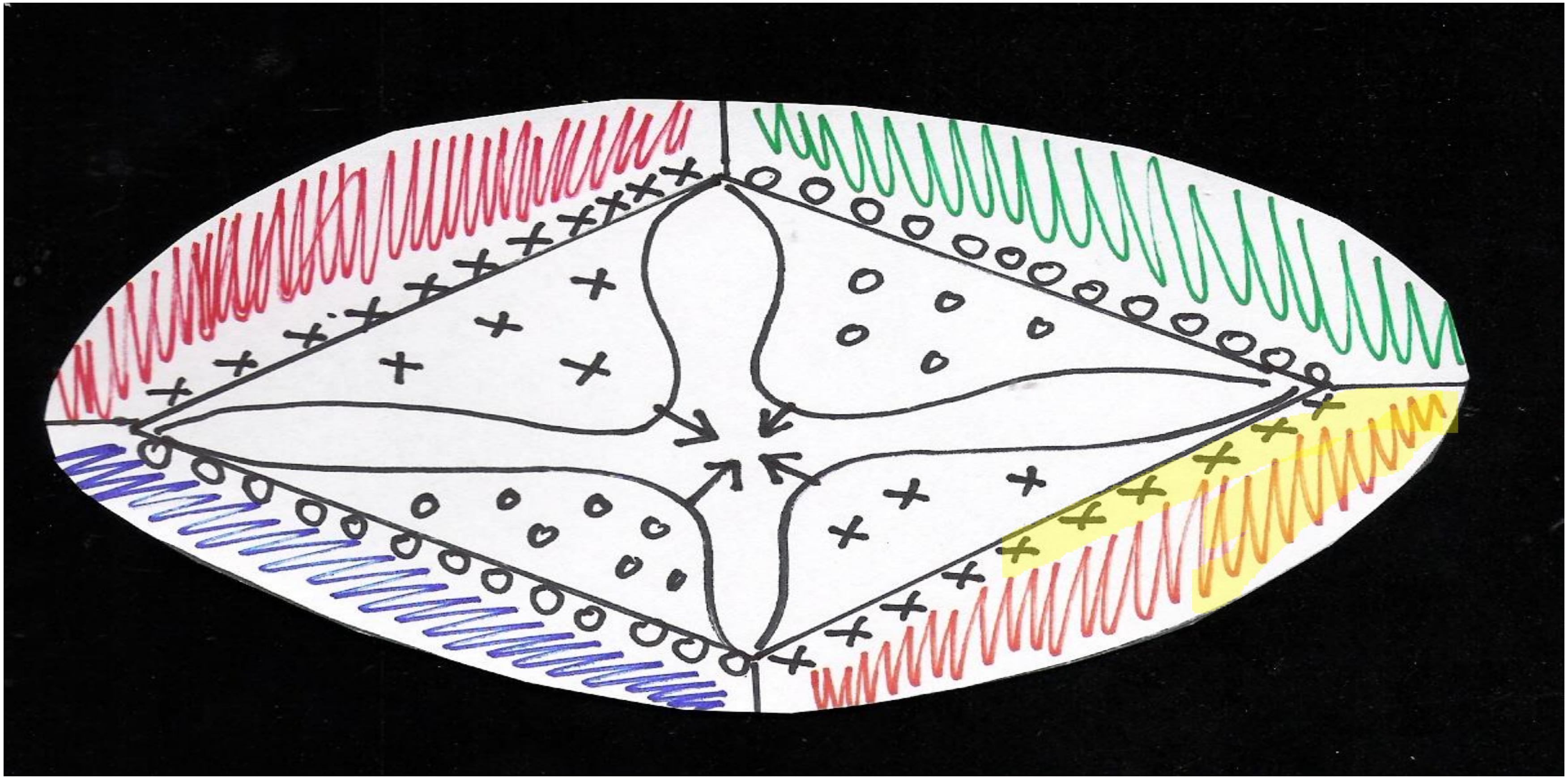


To make a name for learning
when other roads are barred,
take something very easy
and make it very hard.

Piet Hein's two ideas
- two theorems -
creating the HEX game

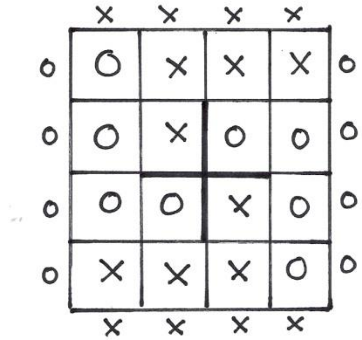
- NOT BOTH CAN WIN
- NOT BOTH CAN LOSE

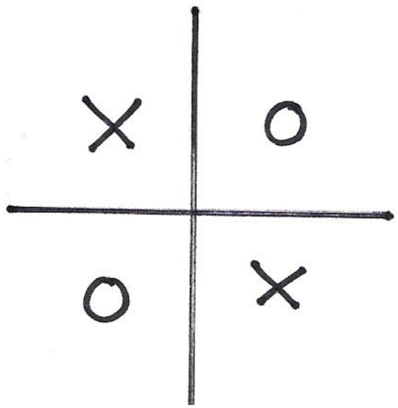
NOT BOTH CAN WIN

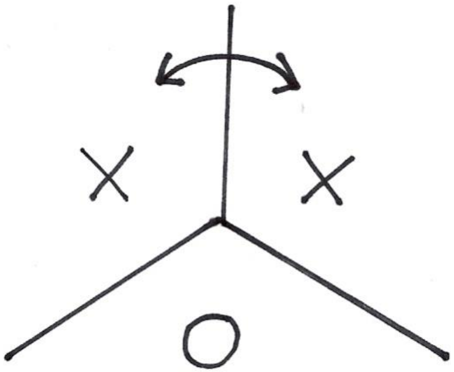


4-COLOUR-THEOREM (1997) : Any map is 4-colourable
SIMPEL SPECIAL CASE: No 5 countries can have common borders two and two

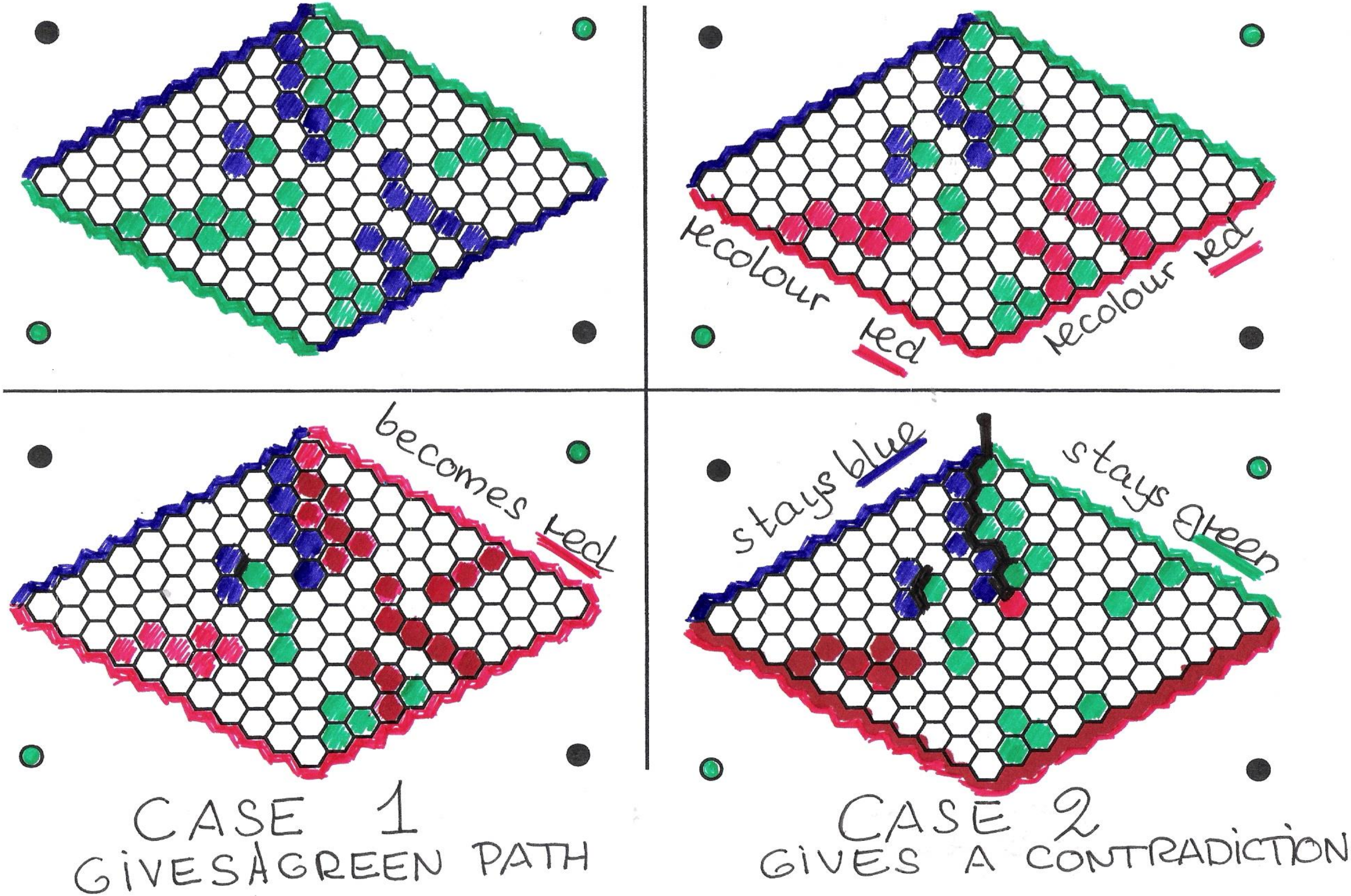
NOT BOTH CAN LOSE





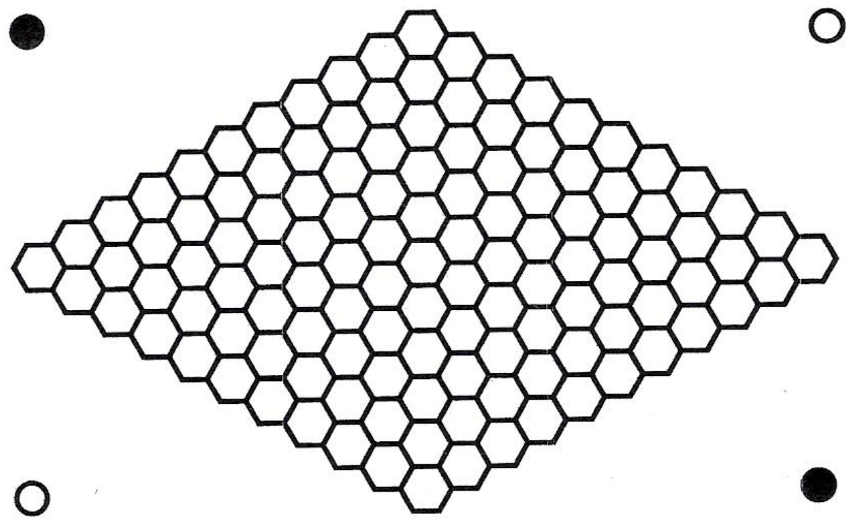


- **PH:** **If only 3 faces meet**
- **PH:** **Then local blocking is impossible**
- **THEN CLEARLY (?):** **Global blocking is impossible**
- **FIRST PUBLISHED PROOF 1969:** **(Anatole Beck et al.)**



The contradiction follows also from
SPERNER's SIMPLEX LEMMA

Piet Hein (1942): **Suddenly in the half-light of dawn a game awoke and demanded to be born**



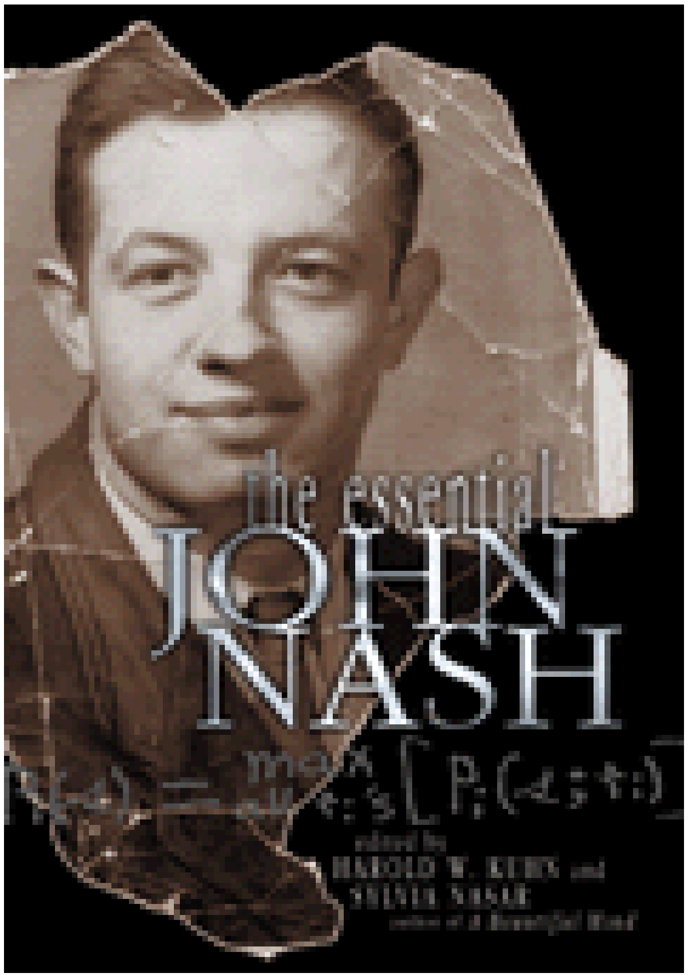
BUT AN ARBITRARY

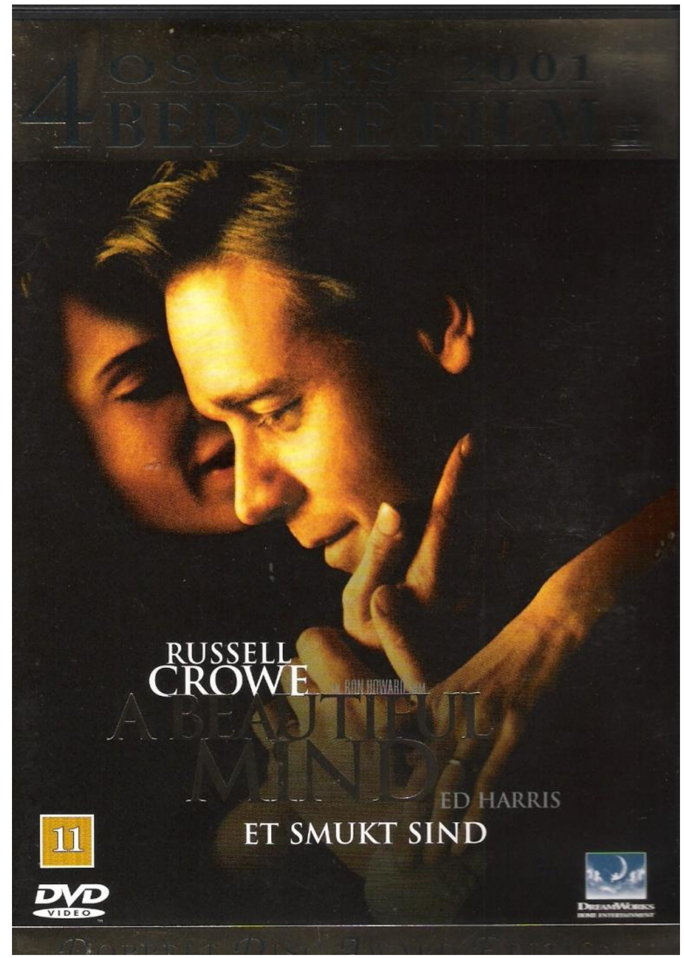
PLANAR 3-REGULAR 2-CONNECTED GRAPH

IS USEABLE AS BOARD (and Piet Hein's Theorems hold)

GENERAL HEX or MUDCRACK HEX

John Nash, 1928-2015, (A Beautiful Mind) discovered Hex in 1948





Non-cooperative Games
John F. Nash Jr. (21 years old)

A DISSERTATION

Presented to the Faculty of Princeton University in Candidacy for the Degree of Doctor of Philosophy

Recommended for Acceptance by the Department of Mathematics

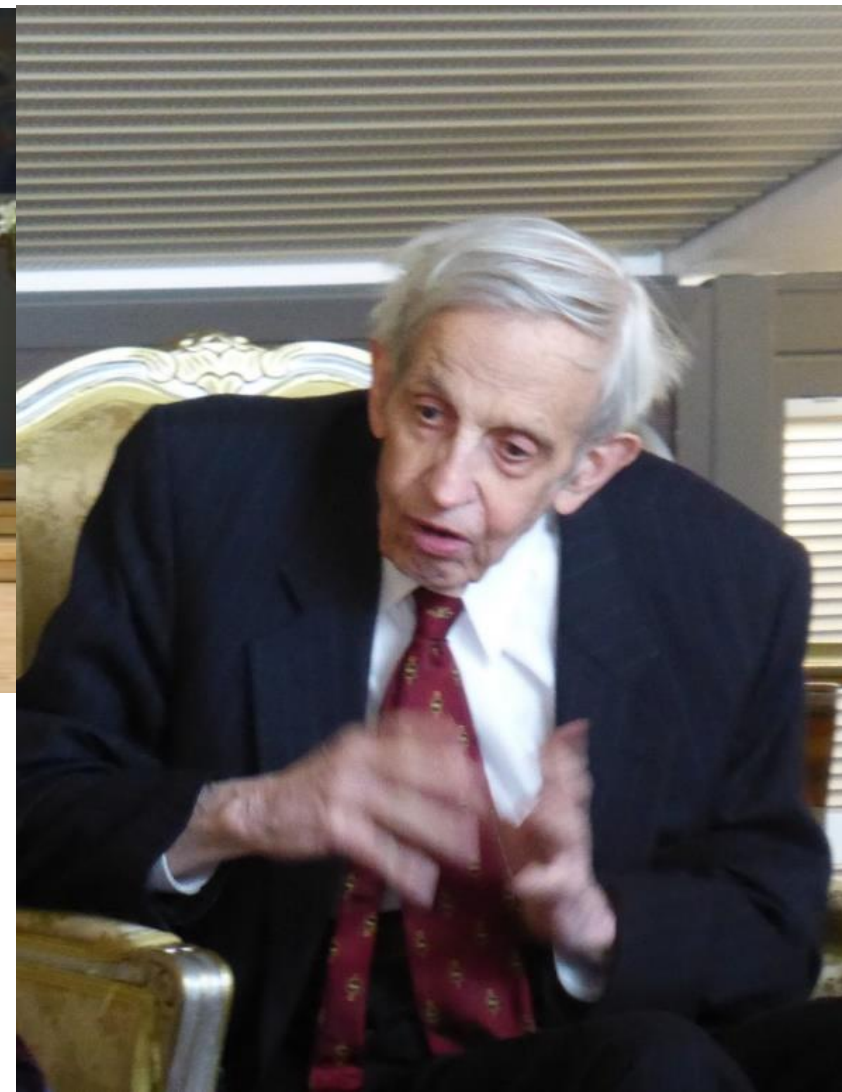
May, 1950

Table of Contents

Stockholm 1994

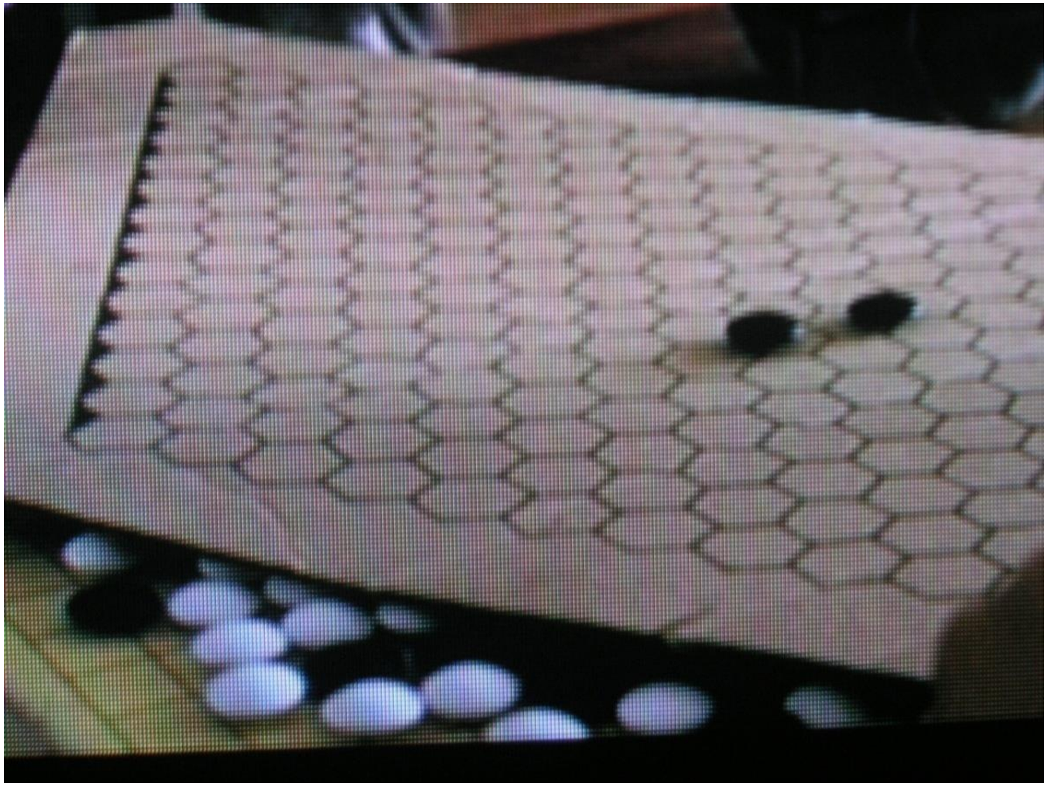
Oslo 2015

The Abel Prize ceremony, May 19th



Deleted scenes from

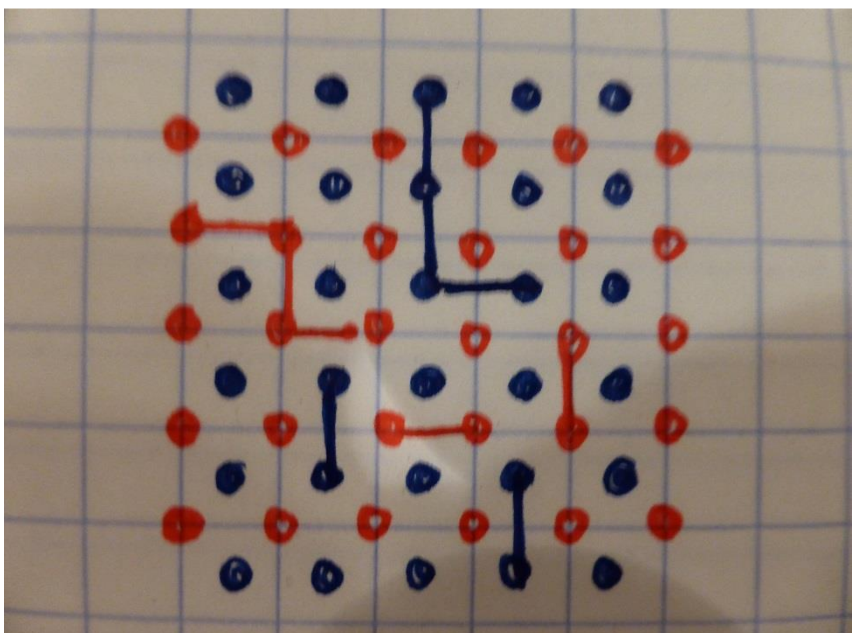
A Beautiful Mind

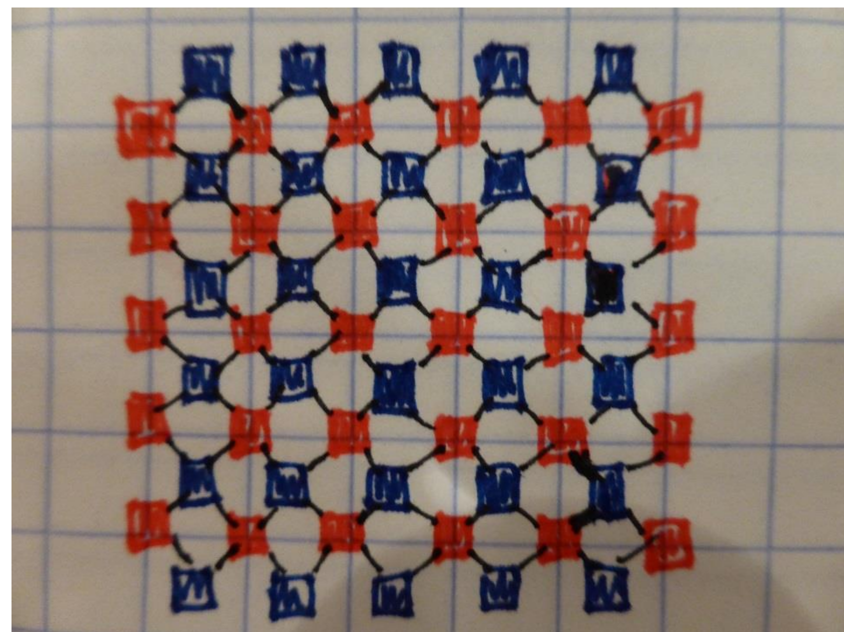


People at Princeton directly involved in discovering/developing/studying Hex

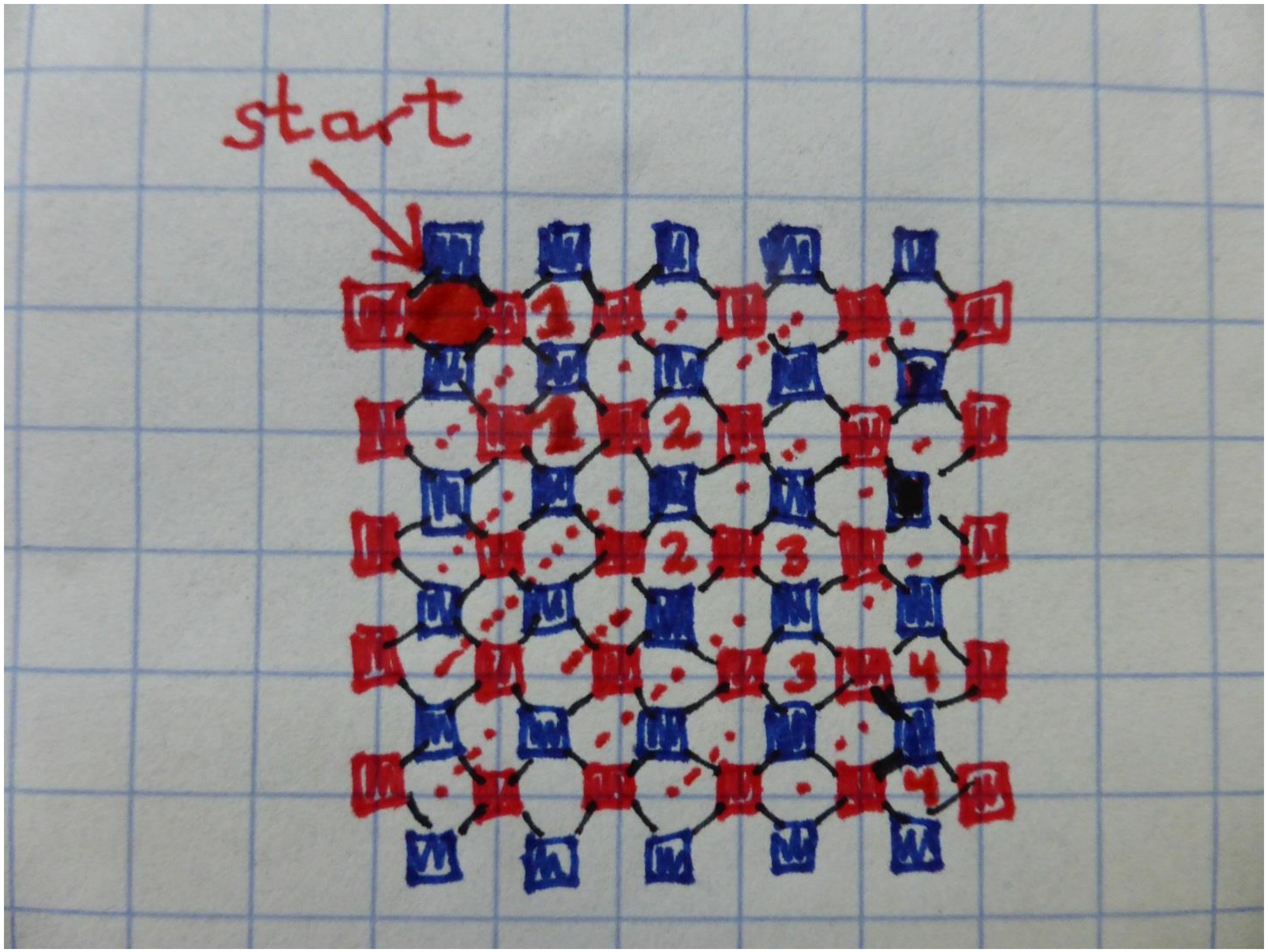
- **John Nash**
- **Aage Bohr**
- **David Gale**
- **Claude Shannon**
- **John Milnor**
- **Harold Kuhn**

David Gale: Bridge-It
Piet Hein: A version of Hex





There is a very simple winning (pairing) strategy for the first player



John Nash's
Hex theorem

- **The first player has a winning strategy**

(but a winning first move for the first player in nxn Hex is **not known with mathematical certainty** – not then and not now!)

Proof: Strategy stealing.

Nash to Gardner 1957

① [illegible] When the board is ~~field~~ filled one ~~or~~ the other of the players will have connected but not both.

② ~~One~~ Either the first player or the second will have a winning strategy.

③ Suppose the second player could force a win.

④ Consider a defensive strategy by first player imitating the winning second player strategy assumed in ③. The first move could be arbitrary. If the strategy ever called for a play where the arbitrary move was made another one could be made.

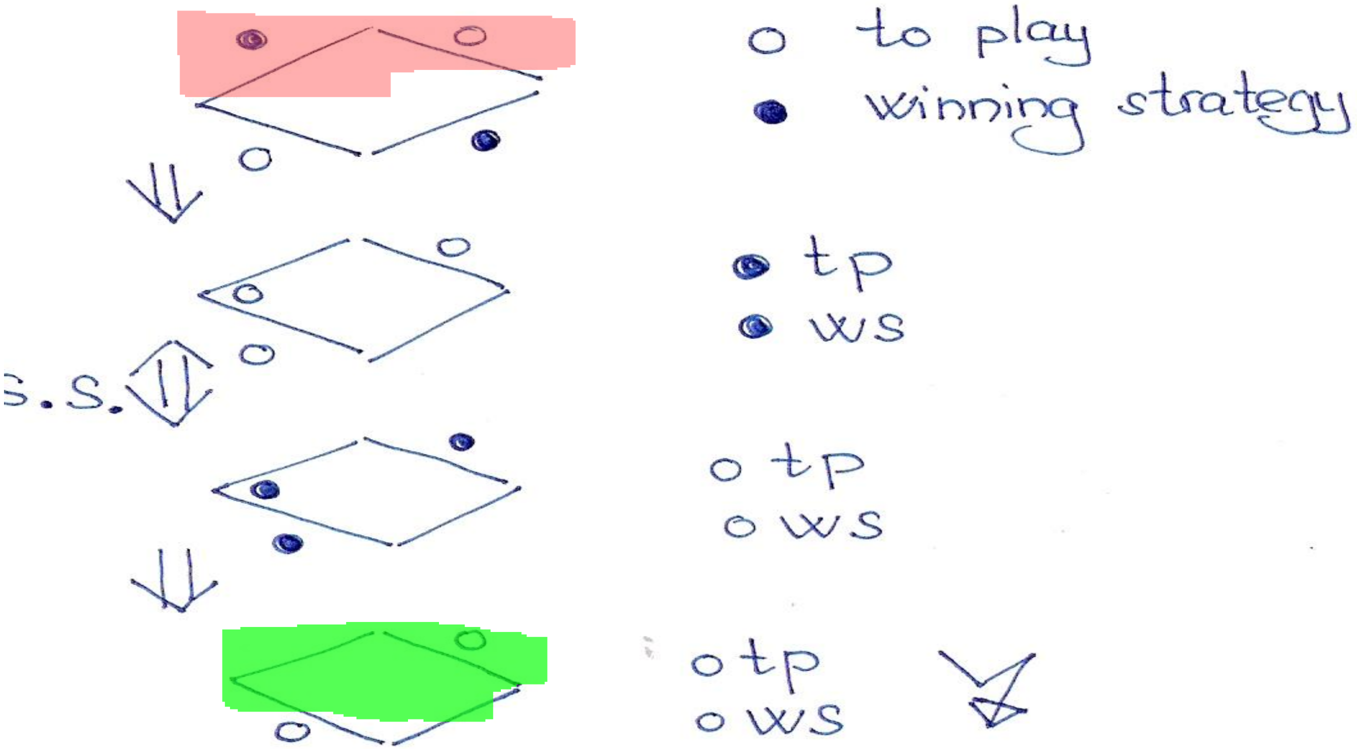
⑤ Since an extra piece on the board is always an asset, never a handicap in connecting, at the end of the game first player will be better off using the adapted (assumed) second player strategy than he would have been if simply playing as second player. So he will win.

⑥ Since this contradicts the hypothesis ③ that second player can win it follows that second player <u>cannot</u> win. Therefore ~~second~~ first player can always win <u>by correct play</u>.

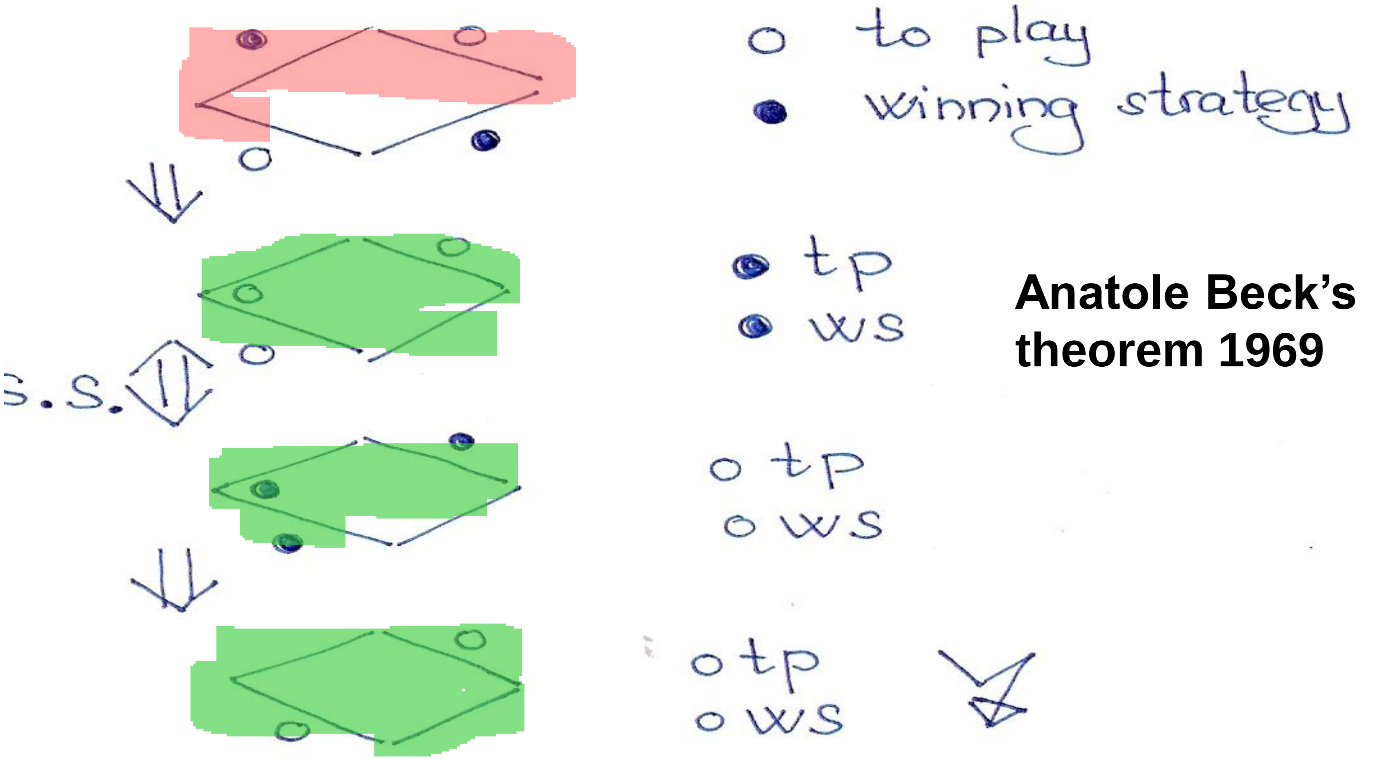
John Nash

173 Bleecker St.
GR 5 4712

Nash's Theorem (strategy stealing)

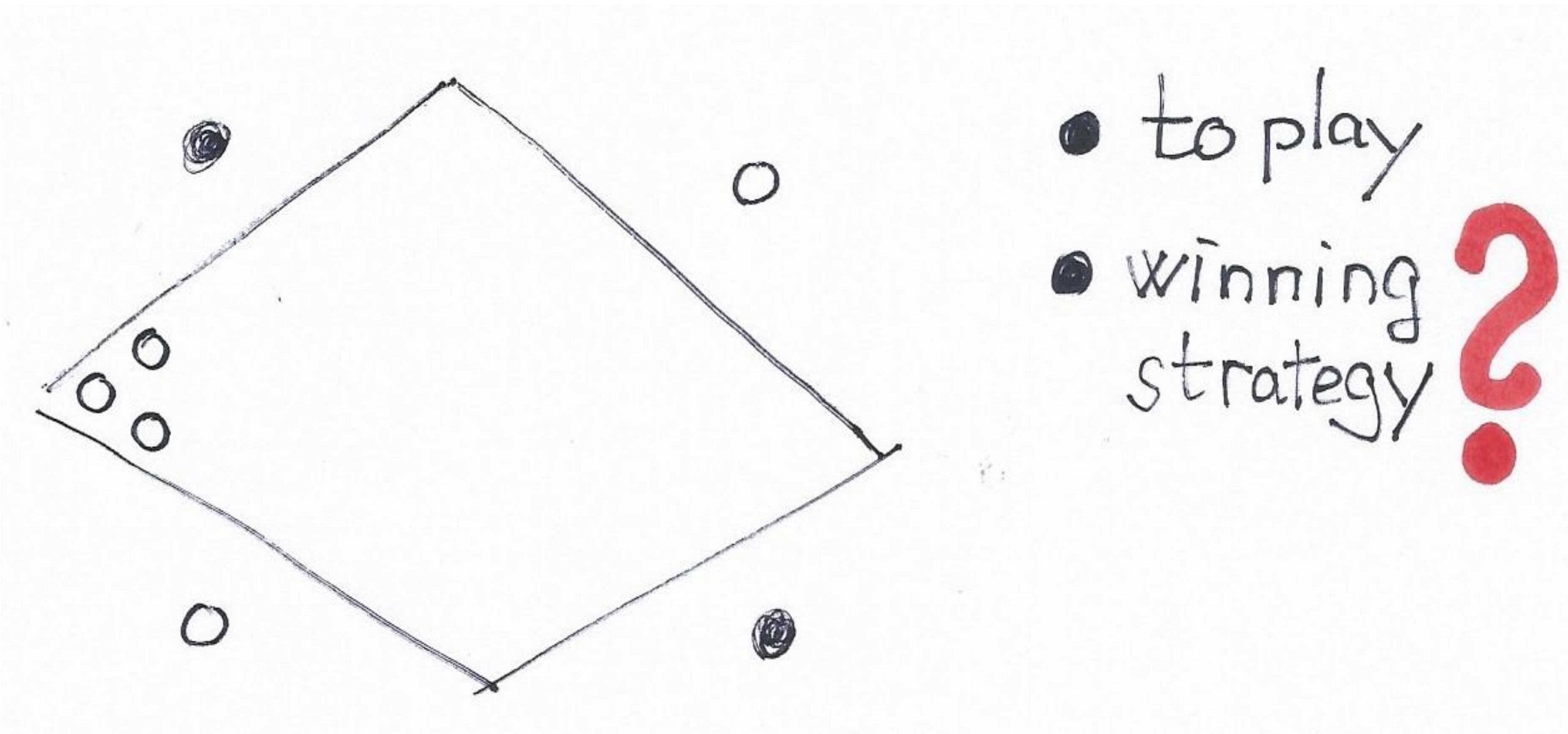


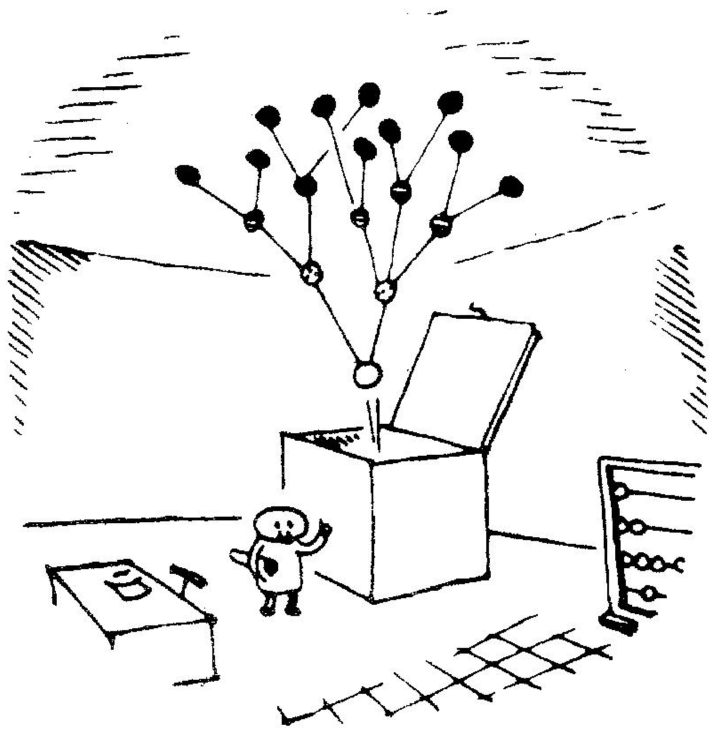
Nash's Theorem (strategy stealing)



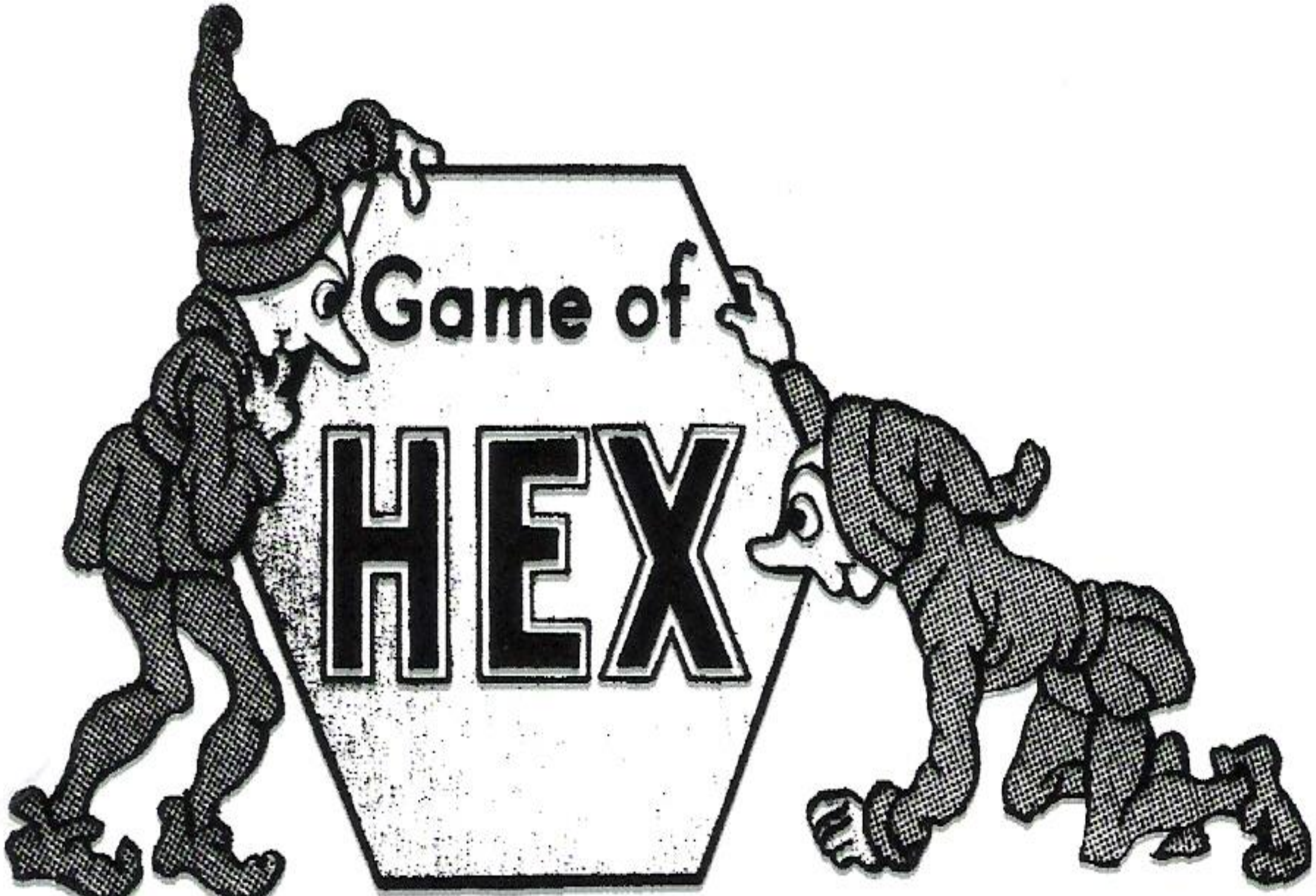
Anatole Beck's theorem 1969

Unsolved problem





We shall have to evolve
problem solvers galore –
since each problem we solve
creates ten problems more.

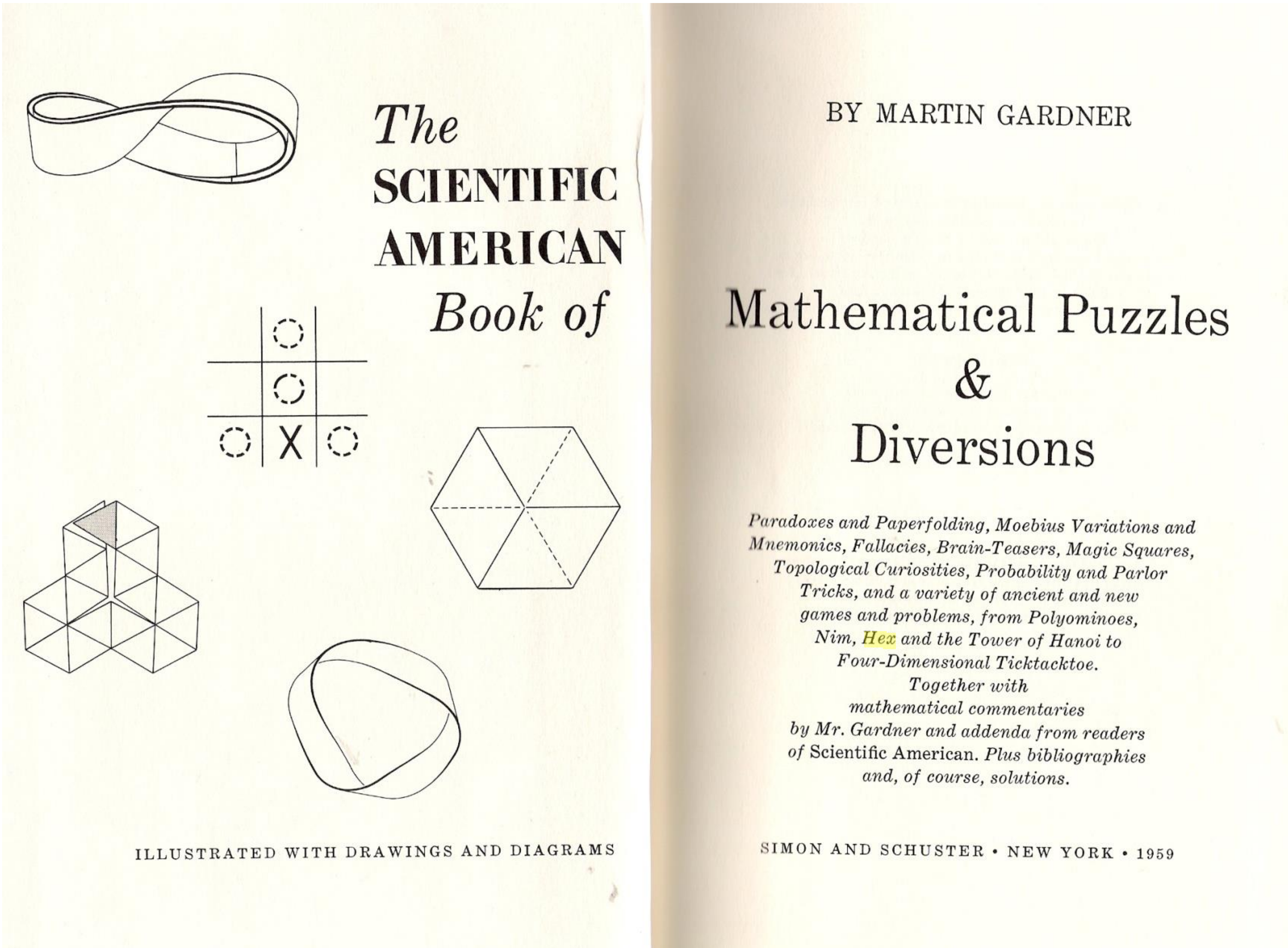


For two players

Parker Brothers Inc.

SALEM, MASSACHUSETTS
NEW YORK CHICAGO
MADE IN U.S.A.

Martin Gardner 1957
SCIENTIFIC AMERICAN



The
SCIENTIFIC AMERICAN
Book of

ILLUSTRATED WITH DRAWINGS AND DIAGRAMS

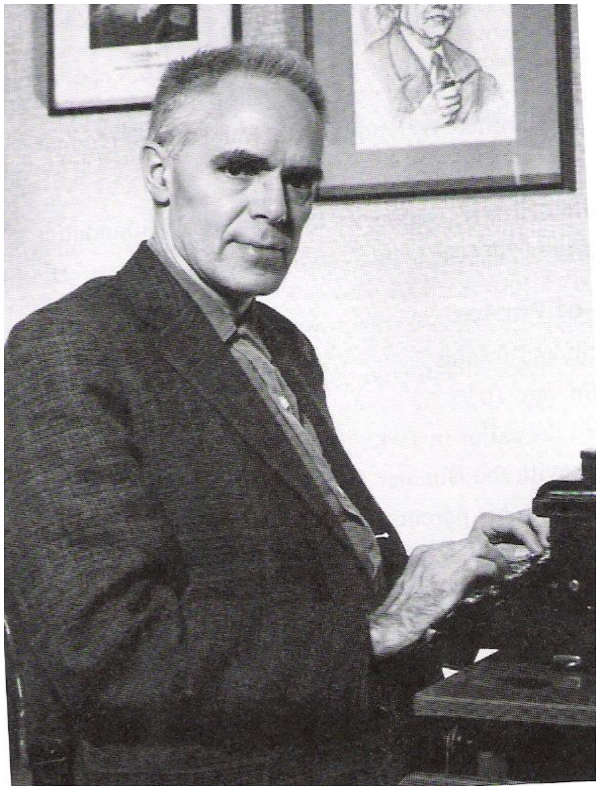
BY MARTIN GARDNER

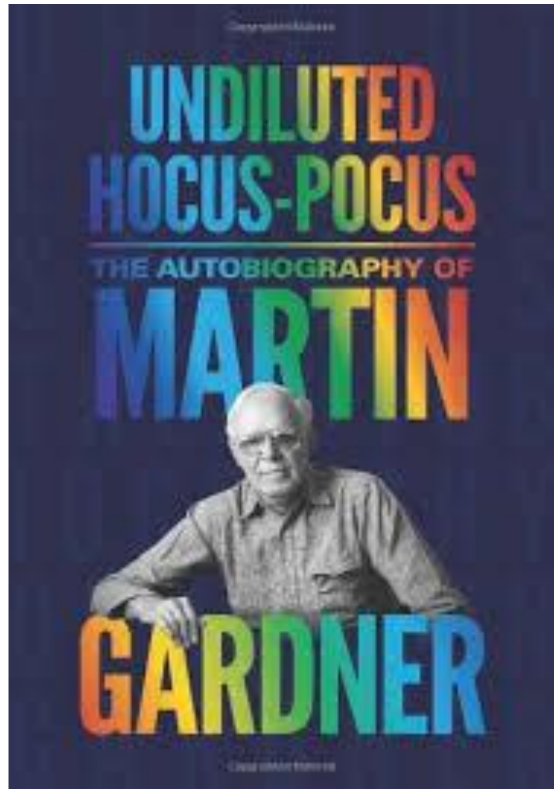
Mathematical Puzzles
&
Diversions

Paradoxes and Paperfolding, Moebius Variations and Mnemonics, Fallacies, Brain-Teasers, Magic Squares, Topological Curiosities, Probability and Parlor Tricks, and a variety of ancient and new games and problems, from Polyominoes, Nim, Hex and the Tower of Hanoi to Four-Dimensional Ticktacktoe. Together with mathematical commentaries by Mr. Gardner and addenda from readers of Scientific American. *Plus bibliographies and, of course, solutions.*

SIMON AND SCHUSTER • NEW YORK • 1959

Martin Gardner 1914-2010





- Piet Hein:
- Black earth turned into
- Yellow Crocus
- Is undiluted
- Hocus pocus

Persi Diaconis: Pick up anything he wrote. You’ll smile and learn something.

Claude Berge playing Hex 1974

Claude Berge
Jean-Marie Pla
Neil Grabois
1974

Claude Berge and Ryan Hayward in Marseilles 1992

Paris juli 2004

Available online at www.sciencedirect.com

SCIENCE DIRECT®

ELSEVIER

Theoretical Computer Science 349 (2005) 123–139

Theoretical Computer Science

www.elsevier.com/locate/tcs

Solving 7×7 Hex with domination, fill-in, and virtual connections☆

Ryan Hayward[a,*], Yngvi Björnsson[b], Michael Johanson[a], Morgan Kan[a], Nathan Po[a], Jack van Rijswijck[a]

[a] *Department of Computing Science, University of Alberta, Edmonton, Alberta, Canada*
[b] *School of Computer Science, Reykjavik University, Iceland*

Abstract

We present an algorithm that determines the outcome of an arbitrary Hex game-state by finding a winning virtual connection for the winning player. Our algorithm recursively searches the game-tree, combining fixed and dynamic game-state virtual connection composition rules to find a winning virtual connection for one of the two players. The search is enhanced by pruning the game-tree according to two new Hex game-state reduction results: under certain conditions, (i) some moves dominate others, and (ii) some board-cells can be "filled-in" without changing the game's outcome.

The algorithm is powerful enough to solve arbitrary 7×7 game-states. In particular, we use it to determine the outcome of a 7×7 Hex game after each of the 49 possible opening moves, in each case finding an explicit proof-tree for the winning player.

Keywords: Hex; Virtual connection; Pattern set; Move ordering; Move domination; Game-state reduction; Fill-in

First winning moves for White

R.B. Hayward, J. van Rijswijck / Discrete Mathematics 306 (2006) 2515–2528

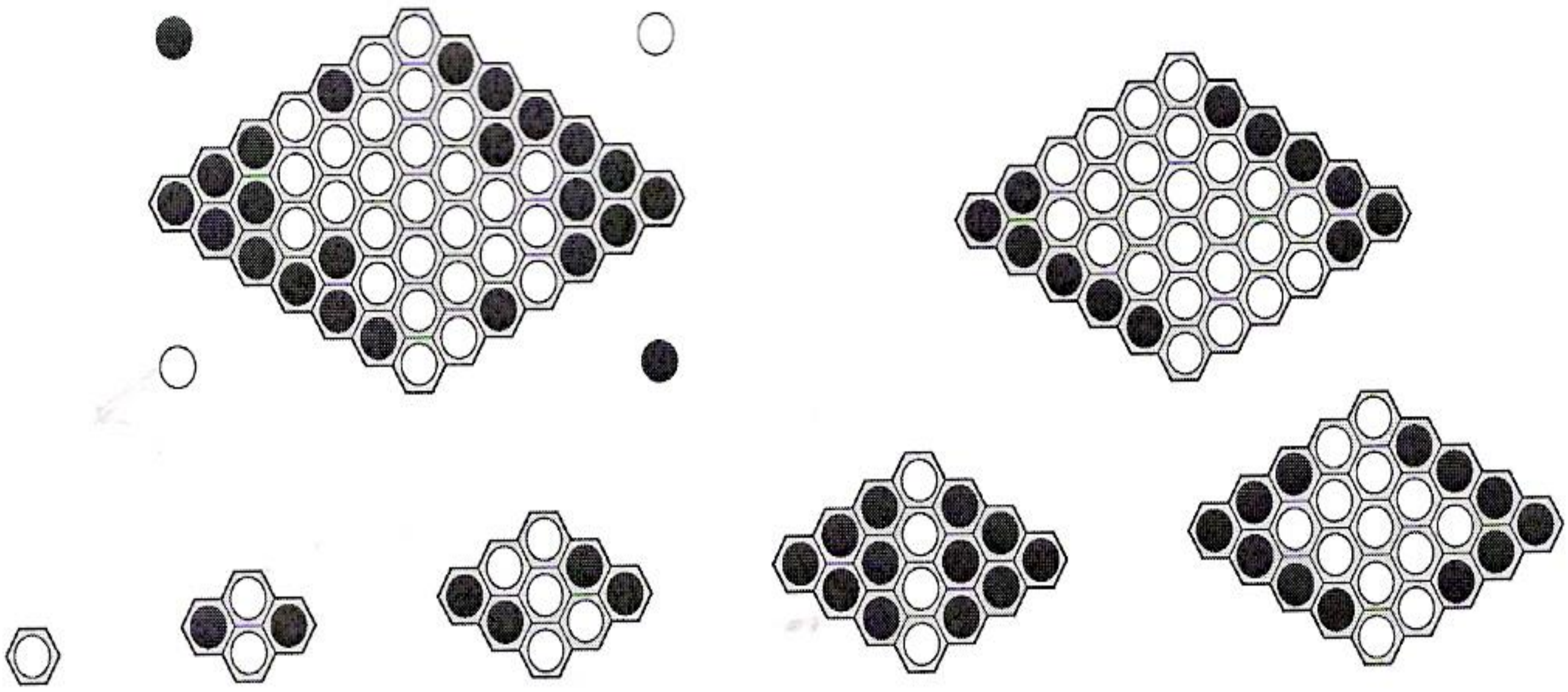


Fig. 7. Winning/losing opening moves. Stone colour indicates the winner if White opens there.

Variation 1: Rex

(Reverse Hex or Misère Hex)

- **Objective: Avoid creating a chain between your two sides!**
- **The game cannot end in draw (hence either the first or the second player has a winning strategy)**
- **On an nxn board with n even the first player has a winning strategy (first published proof: Evans 1974)**
- **On an nxn board with n odd the second player has a winning strategy (first published proof: Lagarias and Sleator 1999). Their proof also covers n even.**

Hayward, Toft and Henderson 2010

How to Play Reverse Hex

Ryan B. Hayward[1], Bjarne Toft[2], and Philip Henderson[3]

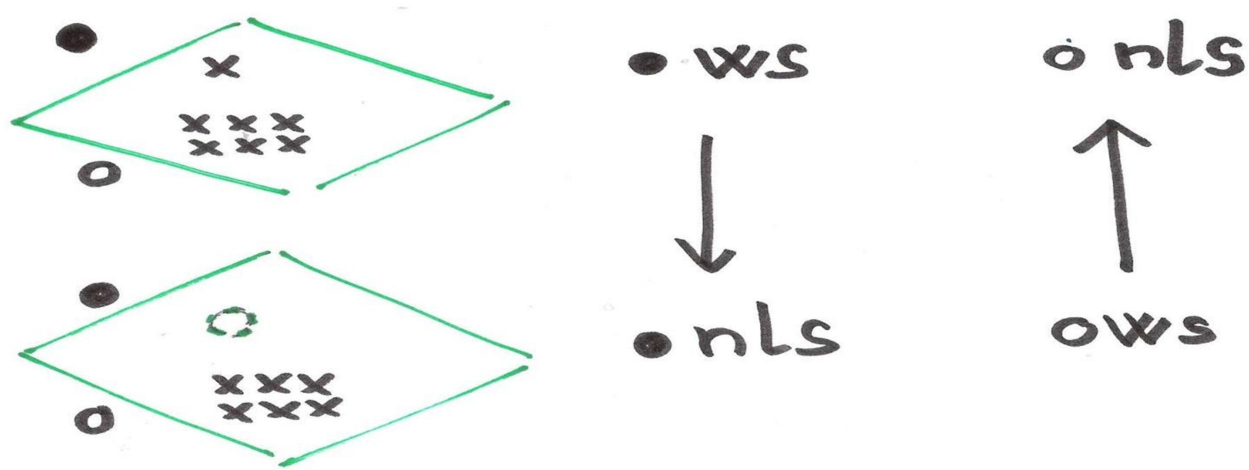
[1] Dept. Comp. Sci., University of Alberta, hayward@ualberta.ca
[2] IMADA, Syddansk Universitet, btoft@imada.sdu.dk
Research supported by NSERC, UofA GAMES, FNU, and IMADA.

Abstract. We present new results on how to play Reverse Hex on $n \times n$ boards. We give new proofs — and strengthened versions — of Lagarias and Sleator's theorem (for $n \times n$ boards, each player can prolong the game until the board is full, so the first/second player can always win if n is even/odd) and Evans's theorem (for even n, opening in the acute corner wins). Also, for even $n \geq 4$, we find another first-player winning opening (adjacent to the acute corner, on the first player's side), and for odd $n \geq 3$ and for each first-player opening, we find a second-player winning reply. Finally, in response to comments by Martin Gardner, we give simple winning strategies for all board sizes up to, and including, 5×5.

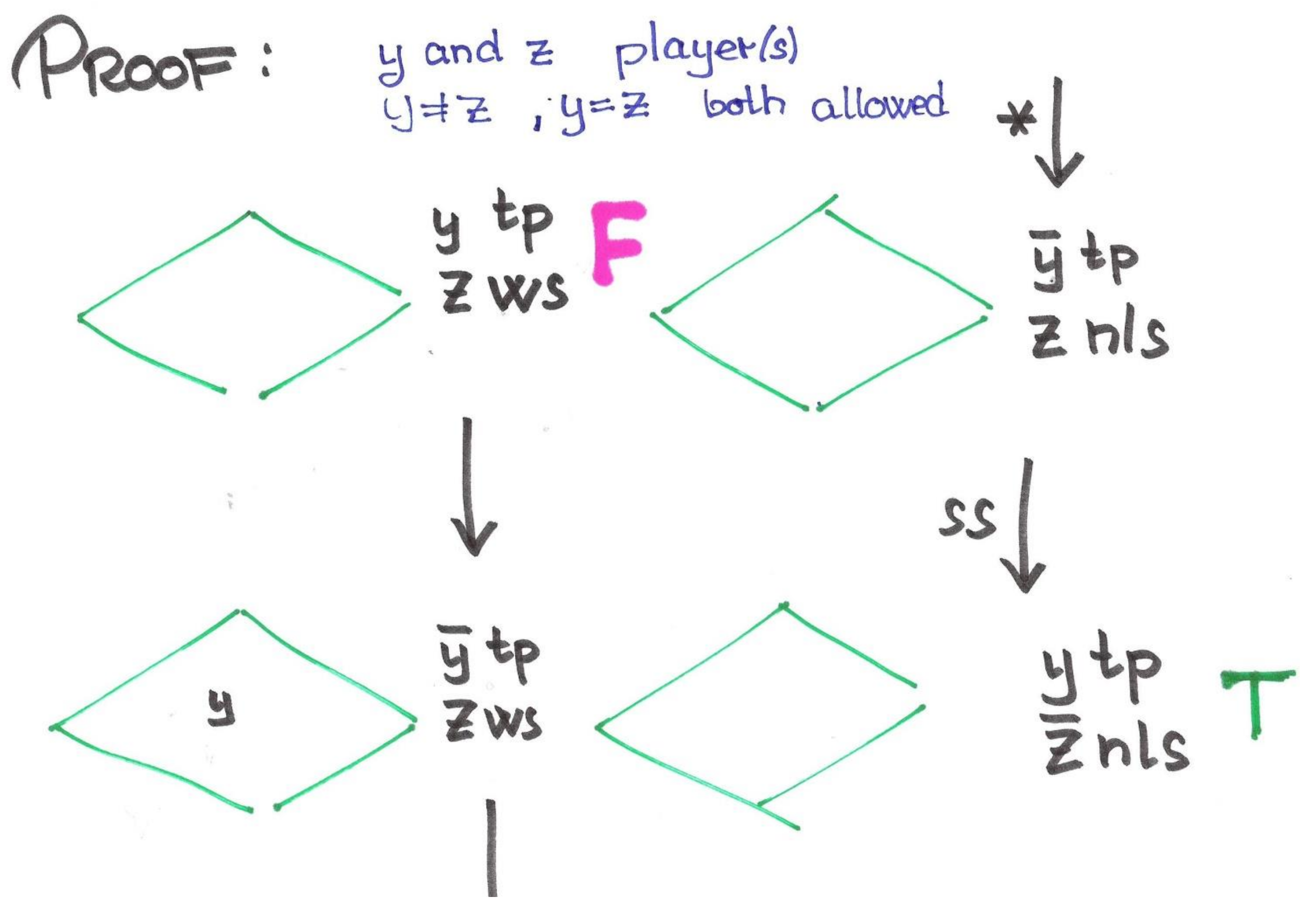
Variation 2: Terminated Rex (TRex) : the Rex game stops when there is just one emty field left (i.e. there should always be a choice!)

LEMMA ✳

ADDING OR REMOVING A PIECE IN TRex CHANGES A WINNING STRATEGY (ws) INTO A NON-LOSING STRATEGY (nls)



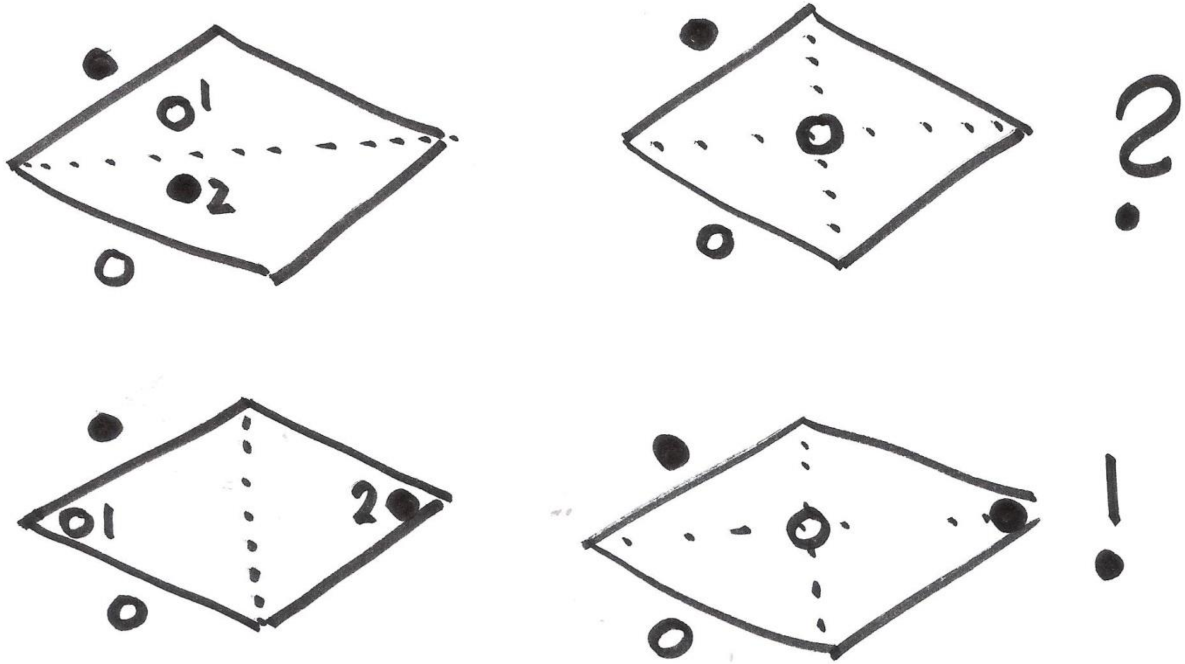
In TRex both players have non-losing strategies



Rex on an nxn-board with n odd:

- Let the second player (Black) play the non-losing strategy from TRex. **THIS IS A WINNING STRATEGY FOR THE SECOND PLAYER IN REX:**
- *Either* the first player (White) creates a white chain *or* TRex ends with one emty field left. In the Rex game that field has to be chosen by White and a White chain is formed!
- If also White plays the non-loosing strategy from TRex, then the Rex game will be decided only when the board is full.

Rex on an nxn board with odd n: (second player has winning strategy)



Variation 3: CYLINDRICAL HEX – play on cylinder!

- THEOREM (Alpern and Belck 1991, Samuel Huneke 2012, Huneke, Hayward and Toft 2014)
- Cylindrical HEX has a winning strategy for the up-down player when the circular dimension n is even (pairing strategy)
- Cylindrical HEX has a winning strategy for the up-down player when the circular dimension is 3
- Problem: Circular dimensions 5, 7, 9, ….?

Discrete Mathematics 331 (2014) 93-97

Discrete Mathematics 331 (2014) 93–97

Contents lists available at ScienceDirect

Discrete Mathematics

journal homepage: www.elsevier.com/locate/disc

A winning strategy for $3 \times n$ Cylindrical Hex

Samuel Clowes Huneke[a,b,*], Ryan Hayward[c], Bjarne Toft[d]

[a] Department of Mathematics, London School of Economics and Political Science, United Kingdom
[b] Department of History, Stanford University, United States
[c] Department of Computer Science, University of Alberta, Canada
[d] IMADA, Syddansk Universitet, Denmark

ARTICLE INFO

Article history:
Received 8 February 2013
Received in revised form 29 April 2014
Accepted 3 May 2014

Keywords:
Hex
Cylindrical Hex
Annular Hex
Steven Alpern
Anatole Beck
David Gale

ABSTRACT

For Cylindrical Hex on a board with circumference 3, we give a winning strategy for the end-to-end player. This is the first known winning strategy for odd circumference at least 3, answering a question of David Gale.

Piet Hein / Bruno Mathsson

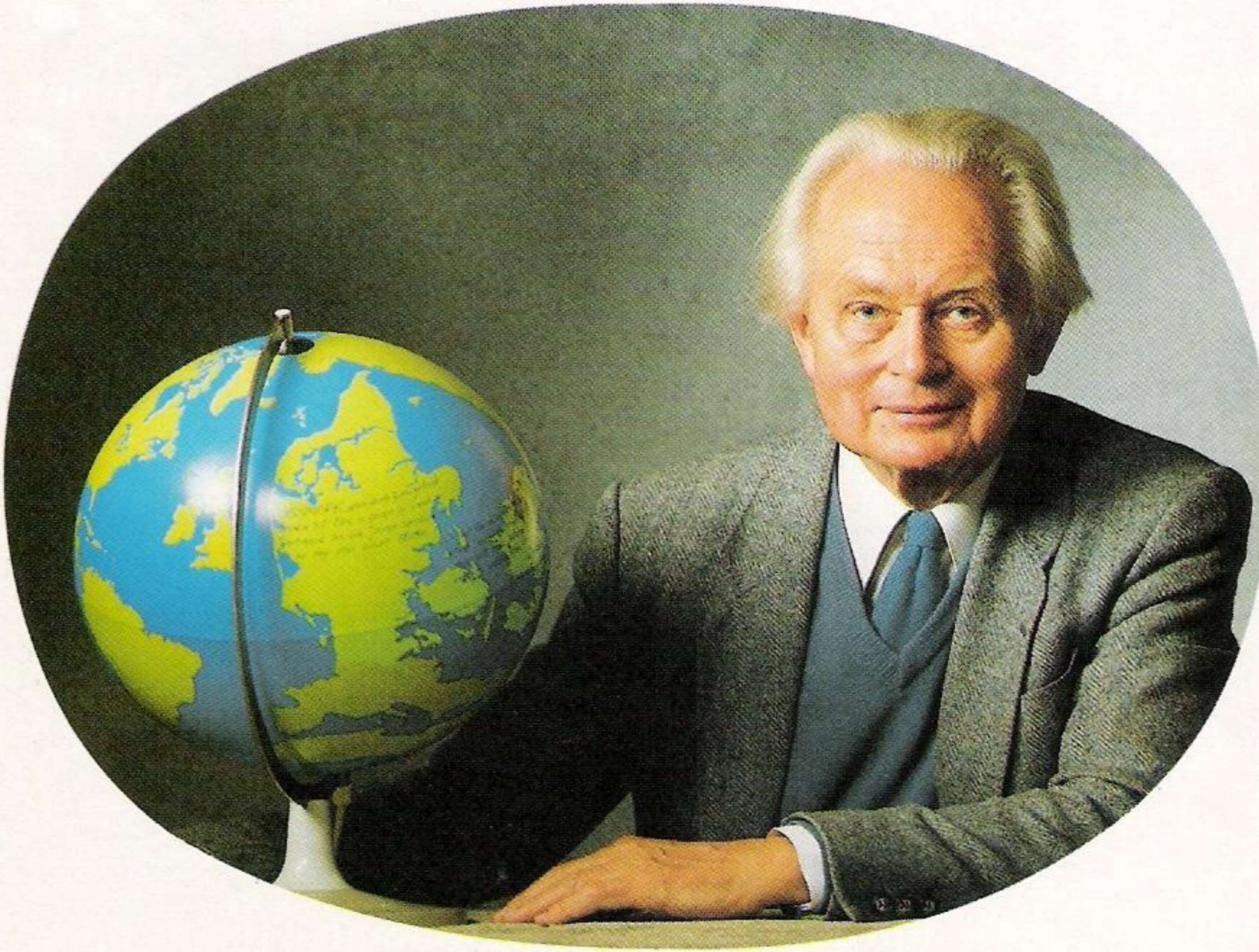
Superegg at Egeskov on Funen, Denmark

Sundial at Egeskov:

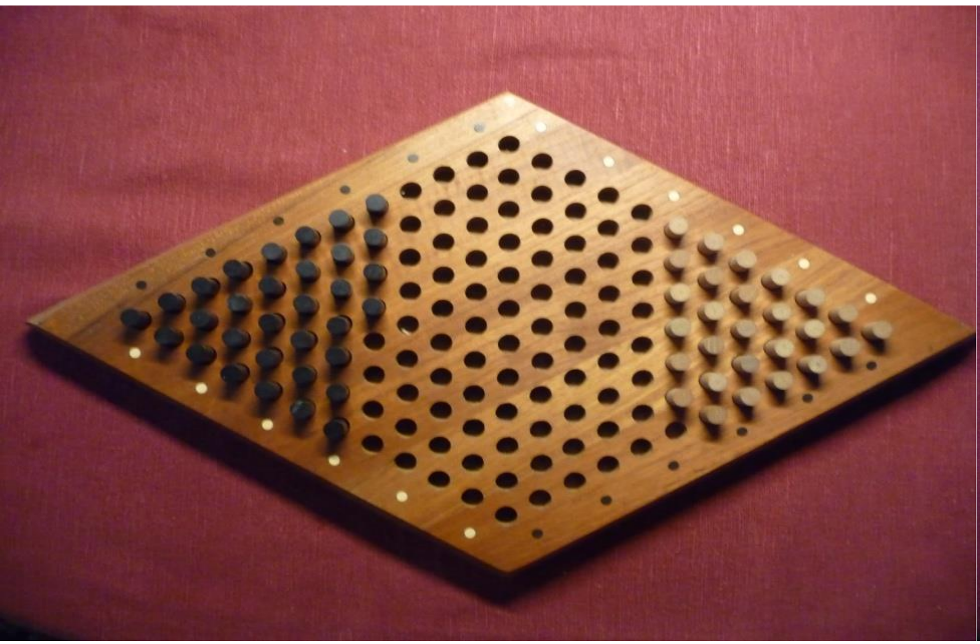
The object casting its shadow, is **the same object** as the one on which the shadow is cast (the screen)!

Piet Hein's new globe of the world

- Denmark seen from foreign land
- Looks but like a grain of sand.
- Denmark as we Danes conceive it
- Is so big you won't believe it.

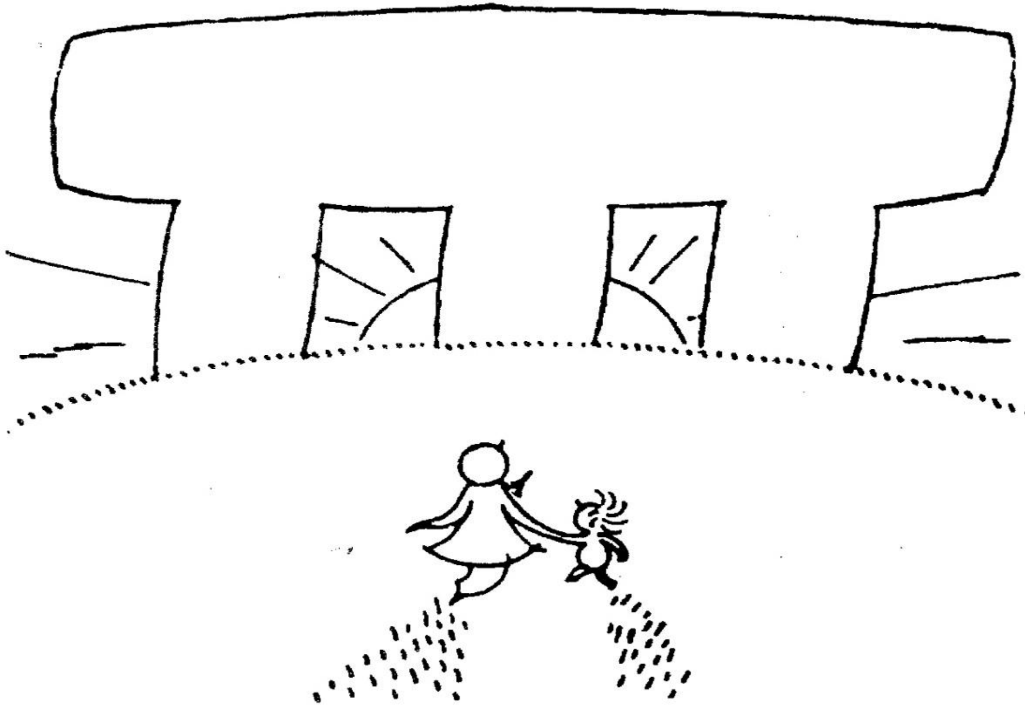


Thank you very much for your attention!



Teak Hex board (by Piet Hein 1968)
Still available at **piethein.com**
Now also as **NEW NORDIC !**
Price around 200 US$

Super elliptic Hex board (by Piet Hein 1975)



Mind these three:
T. T. T.
Hear their chime:
Things Take Time.

Husk de tre:
T. T. T.
Slid men vid:
Ting Tar Tid.